Homework 1
APPM 4720/5720 Spring 2019
Randomized Algorithms

Due date: Friday, Jan 18 2019
Theme: Background

Instructor: Stephen Becker

Instructions   Collaboration with your fellow students is allowed and in fact recommended, although direct copying is not allowed. Please write down the names of the students that you worked with. The internet is allowed for basic tasks only, not for directly looking for solutions.

An arbitrary subset of these questions will be graded.

Problem 1: Skim the first 2 sections of Michael Mahoney's "Randomized algorithms for matrices and data" (2011, part of the Foundations and Trends® in Machine Learning series). DOI link and arXiv link. No response necessary.

Problem 2: Read the "Appetizer" chapter of Roman Vershynin's 2018 "High-Dimensional Probability: An Introduction with Applications in Data Science". Book website and a free PDF on Vershynin's website.

In the last equation of the proof of Theorem 0.0.2 (Approximate Caratheodory's theorem), it is shown that for a random variable Y (where $Y = x - \frac{1}{k}\sum_{j=1}^{k} Z_j$ in Vershynin's notation), it holds that $\mathbb{E}\,\|Y\|_2^2 \le \frac{1}{k}$, and he therefore concludes that there is a realization of $Y = y$ that satisfies $\|y\|_2^2 \le \frac{1}{k}$.

Please turn in a brief response that justifies Vershynin's conclusion. This is a case of Erdös's Probabilistic Method, and you may read online about it, but your response should be in your own words.