APPM 4360/5360 Introduction to Complex Variables and Applications

HOMEWORK #8: For UGs this is all XC (Extra Credit); for Graduate students Problems 1,2 are required but Problems 3,4,5 are XC

Assigned: Wednesday April 24, 2019

DUE: At class Wednesday May 1, 2019

XC: Extra Credit

1. (10 points) 4.5 13a,b

2. (10 points) 4.5 17 a,b,c

3. (10 points) Given the following ordinary differential equation (ODE)

$$\frac{d^3y}{dx^3} + a^3 y = e^{-x}, a > 0$$

with the initial condition: $y(0) = y'(0) = y''(0) = 0$. Assume $y(x)$ has a Laplace transform. Use Laplace transforms to find the solution to the equation. Simplify your result; show that it is real and satisfies the ODE.

4. Given the linear 'free' Schrödinger equation

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} = 0, \quad \text{with} \quad u(x,0) = f(x), \; |x| < \infty$$

Assume that $f(x), u(x,t)$ are Fourier transformable; solve for the Fourier transform of $u(x,t)$; call it: $\hat{U}(k,t)$. Use convolution to find the solution $u(x,t)$ in terms of integrals.

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5. Given the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c > 0$$

with the following initial/boundary conditions

$$u(x, t = 0) = 0, \ \frac{\partial u}{\partial t}(x, t = 0) = 0$$

$$u(x = 0, t) = b, \ u(x = \ell, t) = 0, \quad b > 0, \ell > 0$$

Find the Laplace transform of the solution, call it: $\hat{U}(x, s)$. Then by inverting this Laplace transform find the solution $u(x, t)$ in terms of an infinite sum.