APPM 4360/5360 Introduction to Complex Variables and Applications

HOMEWORK #7

Assigned: Monday April 8, 2019

DUE: At class Wednesday April 24, 2019

XC: Extra Credit

1. Evaluate
$$\int_0^\infty \frac{x^3 \sin kx}{(x^2+a^2)^2} dx, \quad k>0, a>0$$

2. Evaluate
$$\int_0^\infty \frac{\sin kx}{x(x^2+a^2)} dx, \quad k>0, a>0$$

3. Solve 4.3 4, 7a

4. Verify the Argument principle in Theorem 4.4.1 for the functions:

a. $f(z) = \frac{z^3+a^3}{z}, \quad 0<a<1$

b. $f(z) = \text{sech}\pi z$

where the contour is the unit circle: $|z|=1$

5. Solve 4.4.3a

6. Solve 4.4.5a,b

7. Find the Fourier transform of the following functions:

a. . $e^{-x^2+iax}, a>0, \quad$ b. $\frac{x}{x^2+2ax+2a^2}, \ a>0$

8. Find the Inverse Laplace transform of the following functions:

a. $\frac{1}{s^2(s+a)}, \ a>0,$ b. $\frac{1}{(s+b)(s^2+a^2)}, \ a>0, b>0$

9. Given the differential equation for $y(t)$ and initial conditions

$$\frac{d^2y}{dt^2} + \omega^2 y = \cos t, \ y(0)=0, y'(0)=0, \ \omega>0$$

a. Take the Laplace transform of this equation and solve for the Laplace transform of y: $\hat{Y}(s)$

b. Find the inverse Laplace transform of $\hat{Y}(s)$ when $\omega \neq 1$ thereby finding $y(t)$

c. Find the inverse Laplace transform of $\hat{Y}(s)$ when $\omega = 1$ thereby finding $y(t)$

In this way one has solved the differential equation.

XC Solve 4.5.14a,b