

**APPM 4360/5360 Introduction to Complex Variables and Applications**

**HOMEWORK #4**

Assigned: Monday February 18, 2019

**DUE: At class Monday March 4, 2019**

XC: Extra Credit

1. Evaluate the integral  $\oint_C f(z)dz$  where  $C$  is the unit circle (circle centered at the origin, unit radius) enclosing the origin, where  $f(z)$  is given by:

a)  $\log(z + 2)$    b)  $1/(z^2 + 1/4)$

You may use power series representations in problems 2 & 3.

2. Evaluate the integral  $\oint_C f(z)dz$  where  $C$  is the unit circle enclosing the origin, where  $f(z)$  is given below

a)  $f(z) = e^{iz}/z$    b)  $f(z) = (\cos z - 1)/z^3$

3. Solve: 2.6.2 b, c

4. Solve: 2.6.7

5. Discuss whether the sequence  $\{1/(nz)^2\}_{n=1}^{\infty}$  converges and whether the convergence is uniform for:  $0 < \alpha < |z| < 1$ . Discuss whether the convergence is uniform if  $\alpha = 0$

6. Use the set up of problem 3.1.5 for:

a)  $\sum_{n=1}^{\infty} z^{2n}, 0 \leq |z| < R < 1$ ;   b)  $\sum_{n=1}^{\infty} e^{-2nz}, R < \operatorname{Re} z < 1$

7. Find the radius of convergence of the series  $\sum_0^{\infty} a_n(z)$  where  $a_n(z)$  is given by

a)  $(-z^2)^n$    b)  $n^{2n} z^{4n}$

8. Solve 3.2.2 b, d, f

9. Solve 3.2.7

10. Find a series representation for  $1/(1 + z^2)$  for  $|z| > 1$  (Hint: see the discussion and hint of problem 3.2.8)

XC: Solve 2.6.10a,b