**APPM 4360/5360 Introduction to Complex Variables and Applications**

HOMEWORK #3

Assigned: Monday February 11, 2019

**DUE: At class Monday February 18, 2019**

1. a) Given

$$w_1(z) = (z-2)^{1/3}$$

i) Where are the branch points of $w_1(z)$; how many Riemann sheets are associated with $w_1(z)$; explain?

ii) If $z - 2 = re^{i\theta}, -\pi \le \theta < \pi$ find the branch cut associated with $w_1(z)$; explain.

b) Given

$$w_2(z) = \log(z+i)$$

i) Where are the branch points of $w_2(z)$; how many Riemann sheets are associated with $w_2(z)$; explain?

ii) If $z + i = re^{i\theta}, -\pi/2 \le \theta < 3\pi/2$ find the branch cut associated with $w_2(z)$; explain.

2. Find the branch cut structure associated with the function:

$$w(z) = \log(\frac{z-a}{z-b}), \quad a < b, a, b \text{ real}$$

where we use the bipolar coordinates:

$$z - a = r_1 e^{i\theta_1}, \quad z - b = r_2 e^{i\theta_2} \text{ with } 0 \le \theta_1 < 2\pi, 0 \le \theta_2 < 2\pi$$

3. Solve for the bounded solution of Laplace's equation

$$\nabla^2 T = 0$$

in the upper half plane: UHP: $|x| < \infty, y > 0$, with the following boundary conditions given on $y = 0$:

a) $T(x, 0) = \{\alpha \text{ on } x < \ell, \ \beta \text{ on } x > \ell\}$ $\alpha, \beta$ are real constants.

b) $T(x,0) = \{0 \;\text{ on } x < \ell_1, \; \alpha \;\text{ on } \ell_1 < x < \ell_2, \; \beta \;\text{ on } x > \ell_2\}$ $\alpha, \beta$ are real constants.

4. Use the discussion/set up of problem 2.4.1 in the text to evaluate

$$a)1 + z\bar{z}^2 \qquad b)(z-1)/z$$

5. Evaluate $\int_C \frac{1}{z-a} dz$ where $C$ is the unit circle (the unit circle is centered at the origin and has unit radius) for:

a) $|a| < 1$ b)$|a| > 1$; c) What can be said when $|a| = 1$?

6. 2.4 6

7. a) Use the discussion/set up of problem 2.5.2 to evaluate $\int_C \frac{1}{(z-1)(z-3)} dz$;

b) Discuss how to evaluate $\int_C \frac{e^z}{z} dz$ where $C$ is a simple closed contour enclosing the origin; explain your reasoning. Hint: Use eq. 1.2.19 in the text as necessary.

c) Evaluate $\int_C \sqrt{z+2}dz$ where $C$ is the unit circle; explain your reasoning.

XC 2.4.9