APPM 4360/5360 Introduction to Complex Variables and Applications

HOMEWORK #2

Assigned: Monday January 28, 2019

Note: XC: Extra Credit

DUE: At class Monday February 11, 2019

1. Solve 1.3: 6 (Hint: the reverse triangle inequality: $|a-b| \geq |a| - |b|$ is useful.)

2. Discuss the mapping of

a) the upper half z -plane for $f(z) = \overline{f(z)}$ (b) the first quadrant in the z plane for $f(z) = 1/z^2$

(c) Using the stereographic projection discussed in class which maps the z-plane to the sphere whose center is at (0,0,1), south pole is the origin and north pole is (0,0,2) find the points on the sphere which correspond to the complex numbers (i) $z = 1+i$; (ii) $z = x, x$ real; (iii) $z_0 = x + iy$ where x, y lie on the circle $x^2 + y^2 = r^2$; what happens when $r \to \infty$? (iv) On the other hand find the numbers in the complex plane which correspond to the following points on the sphere $(X, Y, Z) = (X, Y, Z = 1)$.

3. Given the function $f(x, y) = \sin x \cosh y + i \cos x \sinh y$. Show whether or not it satisfies the Cauchy-Riemann conditions. If it does find the associated analytic function $f(z)$.

4. In the following the imaginary part of an analytic function is given. Find the real part and the analytic function:

a) $3x^2y - y^3 + k, k = \text{const}$ b) $\frac{-x}{x^2+y^2}$

5. Determine where the following functions are analytic; discuss whether there are any singular points: a) $f(z) = \frac{1}{z^4+1}$ b) $f(z) = \text{cosech} z$ c) $f(z) = e^{\cosh z}$

6. Solve 2.1: 5

7. Find the location and explain why they are branch points for the following functions: a) $(z+i)^{1/3}$ b) $\log \frac{1}{(2z+i)}$

8. Solve for all values of z: $a.$ $7 + 3e^{2z - i\pi} = 4$ b. $\log \frac{3z}{2z+1} = 3i\pi$

9. Derive $\text{cothh}^{-1} z = \frac{1}{2} \log \frac{z+1}{z-1}$ (Hint: use $w = \coth^{-1} z$). Then find $\frac{d}{dz} \coth^{-1} z$

10. Solve 2.2: 7, 8,

11. Solve 2.2.9a

12. Find the location of the branch points and discuss a branch cut structure associated with the functions:

a) $f(z) = (\frac{z}{z+1})^{1/2}$ b) $f(z) = \log(z^2 - 9)$

**XC: Solve : 2.2. 9b,c