APPM 4360/5360 Introduction to Complex Variables and Applications

EXAM #2 Wednesday March 16, 2016

Two pages (8 /2x11) of notes allowed; no calculators or text books

XC: Extra Credit

1. (15) Evaluate the integral $\oint_C f(z)dz$ **where C is the unit circle centered at the origin with:**

a) $f(z) = \frac{\sin z}{z}$

b) $f(z) = \frac{e^{iz}}{z-a}, \quad 0 < a < 1$

2. (15) Suppose $f(z)$ **is an entire function with** $|f(z)| \leq C_1|z|^2$ **and** $|f'(z)| \leq C_2|z|$ **with** $C_1 > 0, C_2 > 0$ **constant. Find the most general form of** $f(z)$**. Explain your reasoning.**

3. (15) Consider $S(z) = \sum_1^\infty e^{-nz}$

a) Show that $S(z)$ **converges uniformly for** $Rez \geq 1$

b) Find the largest region in the z **plane where** $S(z)$ **diverges; explain.**

4. (15) Consider $\quad F(z) = \int_0^\infty f(t)e^{izt}dt$

a) Suppose $f(t)$ **is continuous and** $|f(t)| \leq Ke^{\alpha t}, \ K > 0, \alpha > 0$**. Find the region in the complex plane where** $F(z)$ **is analytic; explain.**

b) Let $f(t) = \frac{e^{-\alpha t^2}}{1+t^3}$ **,** $\alpha > 0$**. Find the region in the complex plane where** $F(z)$ **is analytic; explain.**

5. (20) Find the Laurent series associated with the following functions in the indicated regions

a) $z + 1/z \quad 0 < |z| < \infty$

b) $\frac{z}{z^2+1}, \quad |z| > 1$

c) $\frac{\log(1+z)}{z^2}, \quad |z| < 1$ **, principal branch**

PLEASE TURN OVER

6. (15) Find and classify all singularities and find the the strength of any pole and the residues in the finite z plane for the following functions:

a) $f(z) = \frac{z^2+z+1}{z^2}$

b) $f(z) = \cos(1/z^2)$

c) $f(z) = \cot z$

7. (XC) (10)

A function that has period p satisfies $f(z+p) = f(z)$. Suppose $f(z)$ has periods $p = 1$ and $p = i$ and let $f(z)$ be an entire function. Prove that $f(z)$ must be constant.