Problem 1: (30 points, 6 points each) True/False (answer True if it is always true otherwise answer False).

(a) If $\lambda$ is an eigenvalue of an $n \times n$ matrix $A$, then $1/\lambda$ is an eigenvalue of $A^T$.
(b) The set of eigenvectors belonging to a particular eigenvalue $\lambda$ of an $n \times n$ matrix $A$, together with the zero vector, is a subspace of $\mathbb{R}^n$.
(c) Resonance in a mass-spring system can be prevented by changing the initial velocity of the mass.
(d) The Laplace transform of $f(t) = e^t + e^{-t}$ is $F(s) = \frac{s}{s^2 - 1}$.
(e) In the method of Undetermined Coefficients, the form of the particular solution one should guess for the equation $y'' + 2y' - 3y = t^2 + \sin(t)$ is $y_p(t) = At^2 + Bt + C + D \sin(t) + E \cos(t)$.

Problem 2: (30 points, 10 points each) Short Answer for the following problems. No justification is needed and no partial credit is given.

(a) Compute the inverse Laplace transform of $F(s) = \frac{s}{s^2 + 2s + 2}$.
(b) Let $A$ be a $2 \times 2$ matrix. The eigenvalues of $A$ are $\lambda_1 = 13$ and $\lambda_2 = 31$. What are the eigenvalues of $AA^{-1}$?
(c) Find a basis for the solution space of $x'' - x = 0$.

Problem 3: (30 points) A harmonic oscillator is described by the differential equation

$$x'' + bx' + 4x = f(t)$$

where $f(t)$ is a forcing function and $b$ is a tunable parameter. Answer the following questions:

(a) (10 points) If $f(t) = 0$ and $b = 0$, (i) find the general solution to the homogeneous problem.
   (ii) What is the the period of oscillation?
(b) (10 points) Calculate the value of $b$ such that the harmonic oscillator is critically damped.
(c) (10 points) If $f(t) = F_0 \cos(\omega_ft)$, the particular solution can be written as $x_p(t) = A \cos(\omega_ft - \delta)$. Give the values or conditions for $\omega_f$ such that $|A| \to \infty$ as $b \to 0$. 

**** TEST CONTINUES ON OTHER SIDE OF PAGE ****
**Problem 4:** (30 points) The following questions are unrelated.

(a) (12 points) i) Is \[
\begin{pmatrix}
-1 \\
-1 \\
1
\end{pmatrix}
\] an eigenvector for \[
\begin{bmatrix}
2 & 0 & 0 \\
1 & -1 & -2 \\
-1 & 0 & 1
\end{bmatrix}
\]?

ii) If so, what is the corresponding eigenvalue?

(b) (6 points) Find all eigenvalues of \[
\begin{bmatrix}
1 & -4 \\
4 & 11
\end{bmatrix}.
\]

(c) (12 points) One eigenvalue of \[
A = \begin{bmatrix}
1 & 2 & 2 \\
2 & 0 & 3 \\
2 & 3 & 0
\end{bmatrix}
\] is \(\lambda_1 = 5\).

i) Find the corresponding eigenvector \(\vec{v}_1\).

ii) Another eigenvector of this matrix is \(\vec{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\). Is \(10\vec{v}_2\) also an eigenvector?

**Problem 5:** (30 points) Consider the differential equation \(y'' - 2y' - 3y = f(t)\).

(a) Applying the method of undetermined coefficients, write down the FORM of the particular solution for (do not compute the coefficients):

i) (8 points) \(f(t) = t^2e^{-t}\),

ii) (8 points) \(f(t) = e^{2t}\cos(2t)\).

(b) (14 points) Find the particular solution for \(f(t) = -20\cos(t)\) using the method of undetermined coefficients.