APPM 2360: Midterm exam 3

April 18, 2018

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A letter sized one page crib sheet is allowed.

Laplace transform table

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad \mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}} \qquad \mathcal{L}\{\sin(bt)\} = \frac{b}{s^2+b^2}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2} \qquad \mathcal{L}\{e^{at}\sin(bt)\} = \frac{b}{(s-a)^2+b^2} \qquad \mathcal{L}\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2+b^2}$$

$$\mathcal{L}\{\sinh(bt)\} = \frac{b}{s^2-b^2} \qquad \mathcal{L}\{\cosh(bt)\} = \frac{s}{s^2-b^2}$$

Problem 1: (30 points, 6 points each) **True/False** (answer True if it is always true otherwise answer False).

(a) If λ is an eigenvalue of an $n \times n$ matrix A, then $1/\lambda$ is an eigenvalue of A^T.

(b) The set of eigenvectors belonging to a particular eigenvalue λ of an $n \times n$ matrix A, together with the zero vector, is a subspace of $\mathbb{R}^n$.

(c) Resonance in a mass-spring system can be prevented by changing the initial velocity of the mass.

(d) The Laplace transform of $f(t) = e^t + e^{-t}$ is $F(s) = \frac{s}{s^2-1}$.

(e) In the method of Undetermined Coefficients, the form of the particular solution one should guess for the equation $y'' + 2y' - 3y = t^2 + \sin(t)$ is $y_p(t) = At^2 + Bt + C + D\sin(t) + E\cos(t)$.

Problem 2: (30 points, 10 points each) **Short Answer** for the following problems. No justification is needed and no partial credit is given.

(a) Compute the inverse Laplace transform of

$$F(s) = \frac{s}{s^2+4s+8}.$$

(b) Let A be a 2×2 matrix. The eigenvalues of A are $\lambda_1 = 13$ and $\lambda_2 = 31$. What are the eigenvalues of AA^{-1}?

(c) Find a basis for the solution space of $x'' - x = 0$.

Problem 3: (30 points) A harmonic oscillator is described by the differential equation

$$x'' + bx' + 4x = f(t)$$

where $f(t)$ is a forcing function and b is a tunable parameter. Answer the following questions:

(a) (10 points) If $f(t) = 0$ and $b = 0$, (i) find the general solution to the homogeneous problem. (ii) What is the the period of oscillation?

(b) (10 points) Calculate the value of b such that the harmonic oscillator is critically damped.

(c) (10 points) If $f(t) = F_0\cos(\omega_f t)$, the particular solution can be written as $x_p(t) = A\cos(\omega_f t - \delta)$. Give the values or conditions for ω_f such that $|A| \to \infty$ as $b \to 0$.

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Problem 4: (30 points) The following questions are unrelated.

(a) (12 points) i) Is $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ an eigenvector for $\begin{bmatrix} 2 & 0 & 0 \\ 1 & -1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$?

ii) If so, what is the corresponding eigenvalue?

(b) (6 points) Find all eigenvalues of $\begin{bmatrix} 1 & -4 \\ 4 & 11 \end{bmatrix}$.

(c) (12 points) One eigenvalue of $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$ is $\lambda_1 = 5$.

i) Find the corresponding eigenvector $\vec{v_1}$.

ii) Another eigenvector of this matrix is $\vec{v_2} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$. Is $10\vec{v_2}$ also an eigenvector?

Problem 5: (30 points) Consider the differential equation $y'' - 2y' - 3y = f(t)$.

(a) Applying the method of undetermined coefficients, write down the FORM of the particular solution for (**do not compute the coefficients**):

(i) (8 points) $f(t) = t^2 e^{-t}$,

(ii) (8 points) $f(t) = e^{2t}\cos(2t)$.

(b) (14 points) Find the particular solution for $f(t) = -20\cos(t)$ using the method of undetermined coefficients.