APPM 2360: Midterm exam 2

March 14, 2018

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your recitation section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A letter sized one page crib sheet is allowed.

Problem 1: (30 points, 6 points each) **True/False** (Answer True if it is always true otherwise answer False).

(a) Let $\mathbb{U}_{3\times3} = \{3 \times 3$ upper triangular matrices with real entries$\}$ with the usual matrix operations. The dimension of the space $\mathbb{U}_{3\times3}$ is 9.

(b) If a $n \times n$ matrix A is noninvertible, the system $A\vec{x} = \vec{b}$ has no solutions.

(c) The polynomials $p_1(t) = t + 1$, $p_2(t) = t - 1$, $p_3(t) = 1 + t + t^2$ are linearly independent.

(d) The set $\mathbb{V} = \{2 \times 2$ matrices with zero determinant$\}$, with usual matrix operations, is a vector space.

(e) For any $n \times n$ matrix A, $|AA^T| = |A^T A|$.

Problem 2: (30 points, 10 points each) **Short Answer** for the following problems. No justification is needed.

(a) Calculate all equilibria of the system of differential equations

$$\begin{aligned} x' &= \frac{y^2}{2} - x - 4, \\ y' &= y - x. \end{aligned}$$

(b) Given $B = P^{-1}AP$, where A, B, P are all invertible matrices, find an expression for (i) $|B|$ and (ii) B^{-1} in terms of A and P.

(c) What is a basis for $\text{Span}\left\{ \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \right\}$?

Problem 3: (30 points) question Consider the linear system of equations $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} 3 & k & 0 \\ 0 & 1 & 5 \\ 0 & k & k \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ k \\ 0 \end{bmatrix}$$

(a) (20 points) Solve this system when $k = 0$ using elementary row operations.

(b) (10 points) For which values of k does this system have (i) a unique solution, (ii) no solution, (iii) an infinite number of solutions.

Problem 4: (30 points) In this problem $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & k \\ 0 & 1 & 1 \end{bmatrix}$

(a) (10 points) Compute the determinant of A.

(b) (5 points) Find all possible value(s) of k for which A **is not invertible**.

(c) (15 points) Compute A^{-1}, the inverse of A, for $k = 2$.

Problem 5: (30 points, 10 points each)

(a) For what value(s) of a, if any, can you conclude that the set

$$\{ \sin(at) \ , \ \cos(at) \ , \ 1 \}$$

is linearly independent?

(b) Let $\mathbb{V} = \mathbb{M}_{22} = \{2 \times 2$ matrices with real entries$\}$ and $\mathbb{W} = \{A \in \mathbb{V}$ such that $A^T = -A\}$. Is $\mathbb{W}$ a vector subspace of $\mathbb{V}$? Be sure to fully justify your answer.

(c) Consider the following system of linear equations

$$\begin{aligned} x_1 + 2x_2 + x_3 - x_4 &= 0 \\ x_3 + x_4 &= 0. \end{aligned}$$

(i) Find a basis for the solution space.

(ii) What is the dimension of the solution space?