1. (25 points, Full Answer Problem): Partial credit given, please show all work.
   a. (10 points) Using separation of variables, solve the following equation to find $y(t)$:
   \[ \frac{dy}{dt} = 2ty^2 \]  
   (1)
   b. (5 points) Verify your solution to part (a) by substituting your expression for $y(t)$ back into Equation (1).
   c. (10 points) Suppose that we seek a solution to Equation (1) with the initial condition $y(0) = y_0$. Using this information to eliminate the constant of integration, rewrite your expression from part (a) in terms of $y_0$.

2. (25 points, Full Answer Problem): Partial credit given, please show all work.
   a. (20 points) Find the general solution to
   \[ t^2 \frac{dy}{dt} - y = 1 \]
   using the integrating factor method.
   b. (5 points) If $y_1$ and $y_2$ are both solutions to $y' + 2y = -t$, then is $y_3 = y_1 + y_2$ also a solution to $y' + 2y = -t$?

3. (25 points, Full Answer Problem): Partial credit given, please show all work.
   a. (20 points) Find the general solution of the ODE
   \[ \frac{dy}{dt} = t - y \]
   using the method of variation of parameters (i.e., the Euler-Lagrange method).
   b. (5 points) Solve the IVP with the above differential equation and the initial condition $y(0) = 0$.

TURN OVER
4. (40 points, Short Answer Problem): For the questions in this problem, no motivation is required. If you do submit work, then box your answer, and know that your work will not be graded.

a. (10 points) Consider the initial value problem
\[ y' = y(1 - y)(3 - y), \quad y(0) = 2. \]

Using qualitative analysis, find \( \lim_{t \to \infty} y(t) \).

b. (10 points) The plots in the figure below show the direction fields of the differential equations (i) \( y' = yt \), (ii) \( y' = y + t \), and (iii) \( y' = y - t \) (not necessarily in that order). Which differential equation (i, ii, iii) goes with which plot (a, b, c)?

![Direction Fields](image)

5. (35 points, Full Answer Problem): Partial credit given, please show all work. A tank has a capacity of 50 gal. At the start of an experiment, 4 lb of salt are dissolved in 20 gal of water in the tank. A salt solution with concentration 2 lb/gal is added at a rate of 3 gal/min, and the well-stirred mixture is drained out at the same rate of flow.

a. (15 points) Write down and solve an IVP to find the amount of salt in the tank as a function of time.

b. (6 points) What will be the limiting concentration of salt in the tank (as \( t \to \infty \))? You must use your answer from part (a). Interpret your answer in one sentence.

c. (6 points) How long should this process continue to raise the amount of salt in the tank to 30 lb?

d. (8 points) Now suppose that, at the instant the amount of salt in the tank reaches 30 lbs, the salt solution is turned off and pure water is added to the tank at a rate of 5 gal/min. The well-stirred mixture is drained from the tank at the original rate of 3 gal/min. Write down the new IVP for the rate of change of the amount of salt in the tank (you do not need to solve the IVP).