APPM 2360: Midterm exam 3
November 29, 2017

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your **LECTURE** section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A letter sized crib sheet is allowed.

Problem 1: (30 points) **True/False** (answer True if it is always true otherwise answer False. No justification is needed.)

(a) If all eigenvalues of A are distinct, then A is invertible.
(b) Suppose that for functions $f(t)$ and $g(t)$ we have that their Laplace Transforms $\mathscr{L}[f(t)](s)$ and $\mathscr{L}[g(t)](s)$ exist. Then we know that

$$\mathscr{L}[f(t) \times g(t)](s) = \mathscr{L}[f(t)](s) \times \mathscr{L}[g(t)](s)$$

(where "$\times$" means "multiply").
(c) If A is a 2×2 matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 3$, then the eigenvalues of A^2 are $\lambda_1 = 1$ and $\lambda_2 = 9$.
(d) The solution $x(t)$ of $\ddot{x} + 4\dot{x} + 12x = 0$ with initial values $x(1) = 1$ and $\dot{x}(1) = -1$ is real valued.
(e) In the method of Undetermined Coefficients, the predicted form of the particular solution for the differential equation $\ddot{x} - 6\dot{x} + 9x = e^{3t}$ is $x_p(t) = (A + Bt)e^{3t}$.

Problem 2: (30 points) **Short Answer** for the following problems. No justification is needed.

(a) (7 points) A system is described by the equation $x'' + 3\,x' + 5\,x = \sin(5\,t)$. Find the frequency of the steady-state solution (*i.e.*, the frequency of the solution as $t \to \infty$).
(b) (7 points) A mass of 1 kg is attached to a spring with constant $k = 9$ N/m. There is no damping, i.e., $b = 0$. The system is forced with a forcing term of the form $f(t) = 0.01\cos(\omega_\mathrm{f}\,t)$ (measured in Newtons). Initially the mass is at rest at its equilibrium position. Find a value of ω_f that guarantees that the amplitude of the resulting oscillations grows without limit.
(c) (8 points) Let A be a square matrix, and let $\mathbf{u}_1$ and $\mathbf{u}_2$ be vectors such that $A\,\mathbf{u}_1 = \mathbf{u}_1$ and $A\,\mathbf{u}_2 = \mathbf{u}_2$. Let $\mathbf{w} = \mathbf{u}_1 + \mathbf{u}_2$ and suppose $\mathbf{w} \neq \mathbf{0}$. Is $\mathbf{w}$ an eigenvector of A? If so, what is it's eigenvalue?
(d) (8 points) Let A be a 3×3 matrix with real entries. Which of the following statements could be true for some A:
(A) A has three different complex eigenvalues (with nonzero imaginary part).
(B) A has three different real eigenvalues.
(C) A has two real eigenvalues and one complex eigenvalue (with nonzero imaginary part).

***** TEST CONTINUES ON OTHER SIDE OF PAGE *****

Problem 3: (30 points) Consider the differential equation $y'' - y' - 2y = f(t)$.

(a) Find the general form of the homogeneous solution.

(b) Find the particular solution for $f(t) = 10\sin(t)$ using method of undetermined coefficients.

Write down the FORM of the particular solution for:

(c) $f(t) = 3e^{-t}$, and

(d) $f(t) = -2e^{-t/\pi}$.

DO NOT COMPUTE THE COEFFICIENTS!

Problem 4: (30 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}.$$

(a) Write down the characteristic polynomial that determines eigenvalues of A.

(b) Find the eigenvalues of A.

(c) Find the eigenvector(s) of A or the eigenspace E_λ for its **smallest** eigenvalue. (only the SMALLEST λ!)

Problem 5: (30 points)

(a) (15 points) Starting from its definition, compute the Laplace transform of

$$f(t) = \sinh(2t) + 1 = \frac{e^{2t} - e^{-2t}}{2} + 1.$$

(Please note that providing an answer without derivation will not be counted.)

(b) (15 points) Evaluate the inverse Laplace transform $y(t)$ of

$$Y(s) = \frac{5 - s}{s^2 - 2s + 5}.$$

(A table of Laplace transforms is provided below).

Table 8.1.1 Short table of Laplace transforms

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$	
(i) 1	$\dfrac{1}{s}$	$s > 0$
(ii) t^n	$\dfrac{n!}{s^{n+1}}$	$s > 0$, n a positive integer
(iii) e^{at}	$\dfrac{1}{s-a}$	$s > a$
(iv) $t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$	$s > a$, n a positive integer
(v) $\sin bt$	$\dfrac{b}{s^2+b^2}$	$s > 0$
(vi) $\cos bt$	$\dfrac{s}{s^2+b^2}$	$s > 0$
(vii) $e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s > a$
(viii) $e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s > a$
(ix) $\sinh bt$	$\dfrac{b}{s^2-b^2}$	$s > \lvert b\rvert$
(x) $\cosh bt$	$\dfrac{s}{s^2-b^2}$	$s > \lvert b\rvert$