1. (30 pts) Consider the function 
\[ g(x, y, z) = \frac{4}{49} x^2 - \frac{1}{4} y^2 - z. \]
We want to determine the upward flux of the vector field
\[ \mathbf{F} = -\frac{49}{8} i + 4j + k \]
through that portion of the level surface \( g(x, y, z) = -1 \) lying inside \( \frac{4}{49} x^2 + \frac{1}{4} y^2 = 1. \)

(a) What type of quadric surface is \( g(x, y, z) = -1 \)?
(b) Using an inequality, describe the region \( R \) of the \( xy \)-plane over which you will need to integrate to calculate the requested flux.
(c) Find the integrand you will use to compute the requested flux.
(d) The region \( R \) is kind of messy when it comes to actually computing the flux so let’s consider the change of variables
\[ u = \frac{2x}{7} \quad \text{and} \quad v = \frac{y}{2}. \]
Write an inequality describing the region in the \( uv \)-plane over which the integration will occur under this transformation.
(e) Set up the integral using the order \( dv \, du \) to compute the flux. Don’t evaluate it...yet.
(f) Now, evaluate the integral from part (e) as you see fit. Another change of variables will serve you well in this task.

2. (25 pts) A friend of mine has been wandering aimlessly about \( \mathbb{R}^2 \) starting at the point \((1,0)\) and ending at the point \((0,1)\) in the presence of the force field
\[ \mathbf{F} = \langle Axy - By^3, 4y + 3x^2 - 3xy^2 \rangle \]
noting that no matter what path is taken between these two points the amount of work done is always the same. This nomadic life has been going on a very long time and my friend assures me that every path between the two points has been taken.

(a) What are \( A \) and \( B \)? Briefly explain.
(b) Being in an adventurous yet mathematical mood, I want to know how much work I will do if I walk from \((-1,0)\) to \((3, -2)\) along the path \( y = \sqrt{x+1} \) \((x-2)^{300} (x-4)^{301} \). Can you please tell me?

3. (20 pts) Let \( \mathbf{F} = \langle 3x + \cos y, 2y + \sin z, e^x + 5z \rangle \). Find the outward flux of \( \mathbf{F} \) through the surface enclosing the region inside \( x^2 + z = 1 \), above the \( xy \)-plane and between \( y = 0 \) and \( y = 2 \).

4. (30 pts) Find the circulation of \( \mathbf{V} = i + (x + yz)j + (xy - \cos^2 \sqrt{z}) k \) around the closed path composed of straight lines from \((1, 0, 0)\) to \((0, 0, 2)\) to \((0, 2, 0)\) to \((1, 0, 0)\) by evaluating an appropriate surface integral.

5. (30 pts) Evaluate \( \int_C y^2 \, dx + \left( x^3 + \sqrt{y^3 + 1} \right) \, dy \) where \( C \) is the counterclockwise path consisting of the bottom half of the unit circle with a rectangle of area 1 attached to the top.

6. (15 pts) Daisy the Dachshund is a dog, so she loves to dig in the dirt. In a certain park, the satisfaction that Daisy gets from digging in the dirt at a point \((x, y)\) is given by the function
\[ S(x, y) = x^2 - \frac{1}{3} x^3 - \frac{1}{4} x^4 - 7y^2 \]
Assuming that the park has no boundary, can Daisy find a place in the park to maximize her dirt-digging satisfaction? If so, where is it and what is her maximum satisfaction? If she can’t find a place, explain why not.