On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 7 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work, simplify your answers** Answers with no justification will receive no points. **Please begin each problem on a new page.**
- You will be taking this exam in a proctored and honor code enforced environment. This means: no notes or papers, calculators, cell phones, or electronic devices are permitted.

1. (15 pts) A flat circular plate has the shape of the region \( x^2 + y^2 \leq 1 \). The plate, including the boundary where \( x^2 + y^2 = 1 \), is heated so that the temperature at the point \( (x, y) \) is \( T(x, y) = x^2 + 2y^2 - x \). Find the temperatures and locations of the hottest and coldest points on the plate.

2. (10 pts) Find an equation for the tangent plane to the surface with equation \( zy + y - 1 = xe^z + e^{x+1} \) at the point on the surface where \( x = -1 \) and \( z = 0 \). Put your answer in the form \( ax + by + cz = d \).

3. (15 pts) The mass of a certain object is given by the function \( m = \frac{2}{3} \pi l^3 w^2 \) where \( l \) is the length of the object and \( w \) is its width. Consider two such objects, one with a length of 1 unit and a width of 4 units and another with a length of 4 units and a width of 1 unit. Which object’s mass is more sensitive to a small change in length? Justify your answer using mathematical techniques learned in this course.

4. (15 pts) Find the dimensions of the rectangle of maximum area with sides parallel to the coordinate axes that can be inscribed in the ellipse \( 4x^2 + 16y^2 = 16 \).

5. (20 pts) Jack and Jill went up a hill to the point \( (x_0, y_0, z_0) \) and got caught there in a lightning storm. In a fit of panic Jack darted off in the direction of \( \mathbf{A} = 2 \mathbf{i} - \mathbf{j} \) and noted at that instant that his altitude was changing at an instantaneous rate of \( -\sqrt{5}/5 \) ft/ft. Panic stricken as well, Jill ran in the direction of \( \mathbf{B} = -3 \mathbf{i} + \mathbf{j} \), noting an instantaneous rate of change of altitude of \( -\sqrt{10}/5 \) ft/ft. In what direction should they have run in order to start descending the hill at the fastest rate? What would that rate have been?

6. (15 pts) Find the limit, if it exists, or show that the limit does not exist. Justify your answer.

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\begin{align*}
(a) \quad & \lim_{{(x,y,z) \to (-1/4, \pi/2, 2)}} \tan^{-1} xyz \\
(b) \quad & \lim_{{(x,y) \to (0,0)}} \frac{y \sin x}{x \sin y} \\
(c) \quad & \lim_{{(x,y) \to (0,0)}} \frac{4x^3 y^2}{x^2 + y^2}
\end{align*}
\]

7. (10 pts) Consider the function \( g(x, y, z, t) \) where \( x = u + v^2, \ y = u^2 + v, \ z = \ln(v/u), \ t = e^{uv} \)

Suppose when \( z = -\ln 2 \) and \( v = 1 \) that \( g_x = 4, \ g_y = -3, \ g_z = -6 \) and \( g_t = 2 \). Calculate the instantaneous rate of change of \( g(x, y, z, t) \) with respect to \( u \) at this point.