APPM 2350 **Final Exam** **Spring 2018**

INSTRUCTIONS: Electronic devices, books, and crib sheets are not permitted. Write your name and your instructor's name on the front of your bluebook. Work all problems. Show your work clearly. Note that a correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

Note: this is a Calculus III final exam. While some problems may be solved using techniques from Calculus I or II, you may not receive full credit if you do so, even if the final result is correct.

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1. (40 points) The dome of a planetarium is described by the function $z = \sqrt{900 - x^2 - y^2}$. To service the dome, a very small flat hatch measuring approximately 1 unit by 1 unit is installed tangent to the dome surface. The coordinates of the center of the hatch are $(10, 20, 20)$.

   (a) Determine the unit normal vector to the hatch.

   (b) Determine the standard equation of the plane defined by the hatch.

   (c) If the sun is directly overhead and the hatch is open, approximate the surface area of the patch of sunlight on the floor.

   (d) The air pressure inside the dome is greater than the surrounding atmospheric pressure. As a result, there exists an outward flux of air through the hatch. If the velocity of the air across the hatch is $\mathbf{v} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, estimate the flux of air across the open hatch.

2. (40 Points) On a given day, the temperature distribution (in degrees Fahrenheit) is given by $T(x, y) = 50 + xy + x^2$. Two turtles, Burt and Beatrice, leave their food bowl located at $(0, 0)$ and move off into the first quadrant. Then have had a little argument, so Burt travels along the path defined by $y = x^2$ and Beatrice travels along the path $y = \sqrt{x}$. Each one thinks they travel faster than the other, but they both travel at 2 meters per hour.

   (a) Bert and Beatrice meet again along their paths. Where does this occur?

   (b) Set up, but do not evaluate, the calculations necessary to determine when they meet.

   (c) When they meet, at what rate is $T(x, y)$ changing with respect to *distance* for each turtle? Clearly identify your results.

   (d) If the turtle experiencing the largest rate of increase in temperature with respect to *distance* travels for a short time $\Delta t = 0.1$, by approximately how much will the temperature change for that turtle?

   (e) If the other turtle travels for a short distace $\Delta s = 0.1$, by approximtely how much will the temperature change for that turtle?

3. (40 Points) The strength of a cell phone signal in a certain section of a city can be described by the function $f(x, y) = 9 - \dfrac{x^3}{3} - \dfrac{y^3}{3}$ where $x \geq 0$ and $y \geq 0$. A highway passes through the city along the path described by $xy = 16$. As you drive along the highway, at what location do you get the strongest cell phone signal? The more Calculus 3 concepts you use, the higher your grade!

4. (40 Points) Consider the vector field $\mathbf{F} = -y\,\mathbf{i} + x\,\mathbf{j} + xyz\,\mathbf{k}$ and the finite object bounded on the bottom by the surface $z = x^2 + y^2$, and on the top by $z = 1$.

   (a) Calculate the outward flux of $\mathbf{F}$ across the **top** surface of the object.

   (b) Calculate the outward flux of $\mathbf{F}$ across the **bottom** surface of the object.

   (c) Determine the net outward flux over the surface of the object.

   (d) If possible, verify your "net outward flux" calculation using any theorem(s) from Calculus III, and clearly state your reasoning! Otherwise, clearly write "Cannot be verified".

5. (40 Points) Consider the same vector field and object from the previous problem, and the clockwise path around the "top edge" of the object.

   (a) Calculate the circulation around this path.

   (b) If possible, verify your calculation in part (a) using any theorem(s) from Calculus III, and clearly state your reasoning! Otherwise, clearly write "Cannot be verified."