1. (a) The normal vector to the plane is \( \mathbf{n} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} \) which has magnitude \( ||\mathbf{n}|| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6} \). Thus a unit vector orthogonal to the plane \( P \) is \( \mathbf{n}_P = \frac{1}{\sqrt{6}} (2\mathbf{i} + \mathbf{j} - \mathbf{k}) \).

(b) A vector parallel to the line is given by \( \mathbf{r}'(t) = \mathbf{i} - \mathbf{j} + 3\mathbf{k} \). Letting \( \mathbf{n}_L \) be the vector orthogonal to the line, we need \( \mathbf{r}'(t) \cdot \mathbf{n}_L = 0 \) which will be satisfied if \( \mathbf{n}_L = \mathbf{i} + \mathbf{j} + 0\mathbf{k} \), the magnitude of which is \( \sqrt{2} \). Thus a unit vector orthogonal to \( L \) is \( \mathbf{n}_L = \frac{1}{\sqrt{2}} (\mathbf{i} + \mathbf{j} + 0\mathbf{k}) \).

(c) The parametric equations of the line are

\[
\begin{align*}
x(t) &= t \\
y(t) &= 1 - t \\
z(t) &= 3t
\end{align*}
\]

For the line to intersect the plane, we must find the value of \( t \) that makes these parametric equations satisfy the equation of the plane. Plugging these parametric equations into the equation of the plane gives

\[
2t + (1 - t) - 3t = 3 \implies -2t = 2 \implies t = -1
\]

Using this value in the equation of the line gives

\[
\mathbf{r}(-1) = -\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}
\]

so the point of intersection is \((-1, 2, -3)\).

(d) To find a point in the plane, let \( x = 0 \) and \( y = 4 \). Then using the equation of the plane yields \( z = 1 \). Call this point \( P = (0, 4, 1) \). The point on the line, call it \( S \), is given by \( \mathbf{r}(1) = \mathbf{i} + 0\mathbf{j} + 3\mathbf{k} \) or \( S = (1, 0, 3) \). With this then \( \mathbf{v} = \mathbf{r}'(t) \)

\[
d = \frac{||\mathbf{P\hat{S}} \times \mathbf{v}||}{||\mathbf{v}||} = \frac{||\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 1 & -1 & 3 \end{vmatrix}||}{\sqrt{1^2 + (-1)^2 + 3^2}} = \frac{||-10\mathbf{i} - \mathbf{j} + 3\mathbf{k}||}{\sqrt{11}} = \frac{\sqrt{110}}{\sqrt{11}} = \sqrt{10}
\]

2. (a) \( \mathbf{a}(t) = \mathbf{v}'(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + 0\mathbf{k} \)

(b) We need to find those values of \( t \) with \( 0 \leq t \leq 2\pi \) such that

\[
\mathbf{v}(t) \cdot \mathbf{a}(t) = [\sin t \mathbf{i} + \cos t \mathbf{j} + 0\mathbf{k}] \cdot [(1 - \cos t) \mathbf{i} + \sin t \mathbf{j} + 0\mathbf{k}] = \sin t - \sin t \cos t + \cos t \sin t = \sin t = 0
\]

This occurs if \( t = 0, \pi, 2\pi \).

(c)

\[
\frac{d\mathbf{v}}{ds} = \frac{dt}{ds} \frac{d\mathbf{v}}{dt} = \frac{1}{||\mathbf{v}(t)||} \mathbf{a}(t) = \frac{1}{\sqrt{(1 - \cos t)^2 + \sin^2 t}} (\sin t \mathbf{i} + \cos t \mathbf{j} + 0\mathbf{k})
\]

\[
= \frac{1}{\sqrt{2 - 2\cos t}} (\sin t \mathbf{i} + \cos t \mathbf{j} + 0\mathbf{k})
\]
3. (a) The speed is
\[
\|v(t)\| = \sqrt{t^2 + (2t^3)^2 + (2t^2)^2} = \sqrt{t^2 + 4t^3 + 4t^4} = \sqrt{t^2(1 + 4t + 4t^2)} = t(1 + 2t)
\]
for \(t(1 + 2t) = t(1 + 2t) \text{ since } t \geq 0\)

(b) \[
\frac{dv}{ds} = \frac{dt}{ds} \frac{dv}{dt} = \frac{1}{\|v(t)\|} \frac{dv}{dt} = \frac{1}{t(1 + 2t)} \left( i + 3t^{1/2} j + 4t k \right)
\]

(c) \[
S(t) = \int_0^T \|v(t)\| \, dt = \int_0^T (t + 2t^2) \, dt = \left[ \frac{1}{2} t^2 + \frac{2}{3} t^3 \right]_0^T = \frac{1}{2} T^2 + \frac{2}{3} T^3
\]

(d) \[
r(t) = \int v(t) \, dt + C = \int \left( t i + 2t^{3/2} j + 2t^2 k \right) \, dt = \frac{1}{2} t^2 i + \frac{4}{5} t^{5/2} j + \frac{2}{3} t^3 k + C
\]

With \(r(0) = 0 i + 0 j + 0 k + C = \frac{1}{2} i + \frac{4}{5} j + \frac{2}{3} k\) we have

\[
r(t) = \frac{1}{2} (t^2 + 1) i + \frac{4}{5} (1 + t^{5/2}) j + \frac{2}{3} (1 + t^3) k
\]

4. (a) \(v(t) = r'(t) = -\sin t \, i + \cos t \, j + \sin t \, k\)

(b) \(\|v(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (\sin t)^2} = \sqrt{1 + \sin^2 t}. \text{ For } t \geq 0, 0 \leq \sin^2 t \leq 1 \implies 1 \leq \|v(t)\| \leq \sqrt{2}. \text{ Thus minimum speed is } 1 \text{ and maximum is } \sqrt{2}.\)

(c) The curve’s shadow is parameterized as \(s(t) = \cos t \, i + \sin t \, j + 0 \, k\) so that \(x(t) = \cos t \text{ and } y(t) = \sin t\). From this we have \(\dot{x}(t) = -\sin t, \dot{y}(t) = -\cos t, \dot{y}(t) = \cos t\). Then

\[
\kappa = \frac{|\dot{x} \ddot{y} - \ddot{x} \dot{y}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}} = \frac{|(-\sin t)(-\sin t) - (\cos t)(-\cos t)|}{|(-\sin t)^2 + (\cos t)^2|^{3/2}} = 1
\]

(d) When \(t = \frac{3\pi}{2}\), we are at the point \(r(3\pi/2) = \cos(3\pi/2) \, i + \sin(3\pi/2) \, j + [1 - \cos(3\pi/2)] \, k = 0 i - j + k\). At that point, we are looking in the direction of the tangent vector, which is \(v(3\pi/2) = -\sin(3\pi/2) \, i + \cos(3\pi/2) \, j + \sin(3\pi/2) \, k = i + 0 j - k\). The tangent line at that point (direction we are gazing) is given by

\[
L(\tau) = \langle 0, -1, 1 \rangle + \tau \langle 1, 0, -1 \rangle = \langle \tau, -1, 1 - \tau \rangle, \quad -\infty < \tau < -\infty
\]

The coordinates of the point on the ground at which we are looking are found by determining the intersection of this tangent line with the ground, that is, setting the \(z\) coordinate to zero. Doing so yields \(z = 1 - \tau = 0 \implies \tau = 1\). Thus \(L(1) = (1, -1, 0)\) so the coordinates of the point on the ground at which we are looking are \((1, -1, 0)\).

5. (a) \(r(t) = (1, 2, 3) + t(6 - 1, 5 - 2, 4 - 3) = (1 + 5t) \, i + (2 + 3t) \, j + (3 + t) \, k, \quad -\infty < t < -\infty\)

(b) \(r(t) = 3i + t^2 j + t k, \quad -\infty < t < -\infty\)

(c) \(r(t) = 3 \cos t \, i + 4j + \sin t \, k, \quad 0 \leq t \leq 2\pi\)
(d) The curve of intersection is found by setting the equations describing the two surfaces equal to one another, giving

\[ 2 - x^2 - y^2 = y^2 - x^2/2 \implies 2 = 2y^2 + \frac{1}{2}x^2 \implies \frac{x^2}{4} + y^2 = 1 \]

which is an ellipse in the \(xy\)-plane. This is parameterized as \(r(t) = 2 \cos t \mathbf{i} + \sin t \mathbf{j}\). The \(z\)-coordinate of the intersection curve is obtained by using these values of \(x\) and \(y\) in the equation of either surface, yielding

\[
\begin{align*}
    r(t) &= 2 \cos t \mathbf{i} + \sin t \mathbf{j} + (\sin^2 t - 2 \cos^2 t) \mathbf{k} \\
    &= 2 \cos t \mathbf{i} + \sin t \mathbf{j} + (2 - 4 \cos^2 t - \sin^2 t) \mathbf{k}
\end{align*}
\]