1. (25 points)
Consider the path given by \( \mathbf{r}(t) = x(t) \mathbf{i} + y(t) \mathbf{j} \) for \( t \geq 0 \) corresponding to the curve \( y = x^3 \) in the \( x-y \) plane.

(a) Determine two functions \( x_1(t) \) and \( y_1(t) \) to yield a parametrization of the path \( \mathbf{r}_1(t) \) for \( t \geq 0 \).

(b) Determine two more functions \( x_2(t) \) and \( y_2(t) \) to yield a second, and different, parametrization of the path \( \mathbf{r}_2(t) \) for \( t \geq 0 \). Note: it is the same path, just two different parametrizations.

(c) Determine the unit tangent, \( \mathbf{T} \), to the path at the point \((2, 8)\). Write your result in terms of the unit vectors \( \mathbf{i} \) and \( \mathbf{j} \).

(d) Determine the unit normal, \( \mathbf{N} \), to the path at the point \((2, 8)\). Write your result in terms of the unit vectors \( \mathbf{i} \) and \( \mathbf{j} \).

(e) Determine the curvature of the path at the point \((2, 8)\).

2. (25 points) Consider the two planes described by \( x + 2y + 3z = 14 \) and \( 3x + 2y + z = 10 \).

(a) Is the point \((1, 2, 3)\) on each of these the planes?

(b) Determine a direction vector, \( \mathbf{D} = a \mathbf{i} + b \mathbf{j} + c \mathbf{k} \), where \( a, b, \) and \( c \) are all integers, for the intersection line.

(c) Using \( \mathbf{D} \), determine a parameterization of the intersection line of these planes.

(d) What is the closest distance between the origin and the intersection line?

3. (25 points)
It is fall and Simone the squirrel is out looking for any remaining nuts when suddenly she spots a kitty cat. To avoid the kitty cat Simone suddenly runs along the path \( \mathbf{r}(t) = (\sin t - t \cos t) \mathbf{i} + t^2 \mathbf{j} + (\cos t + t \sin t) \mathbf{k} \), for \( t \geq 0 \).

(a) Determine Simone’s velocity \( \mathbf{v}(t) \), speed \( |\mathbf{v}(t)| \), and acceleration \( \mathbf{a}(t) \).

(b) Is Simone’s acceleration \( \mathbf{a}(t) \) ever orthogonal to her velocity \( \mathbf{v}(t) \)? If so, when?

(c) If Simone runs for \( 0 \leq t \leq 2 \), how far has she traveled? Note: this is arc length, not displacement!

4. (25 points) Classify (sphere, ellipsoid, elliptic cylinder, elliptic paraboloid, nothing I’ve ever heard of, etc.) for each of the following surfaces. No supporting work or justification is necessary. Note: choose your words carefully — for example, distinguish between an ellipse and a circle, centered on the \( x \)-axis, and so on.

(a) \( xy = 9 \)

(b) \( 8x^2 - 5y^2 - 7z^2 = 6 \)

(c) \( 24x^2 - y^2 + 16z^2 = 0 \)

(d) \( 6x^2 + 2y + 6z^2 = 0 \)

(e) \( 3x^2 - 3y^2 - 3z = -5 \)
Projections, distances from point $S$ to line containing point $P$, and $S$ to plane with normal $n$

$$\text{proj}_A B = \left( \frac{A \cdot B}{A \cdot A} \right) A \quad d = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|} \quad d = \frac{|\vec{PS} \cdot \vec{n}|}{|\vec{n}|}$$

Arc length, Frenet formulas, and tangential and normal acceleration components

$$ds = |\vec{v}| \, dt \quad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\vec{v}}{|\vec{v}|} \quad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \quad \mathbf{B} = \mathbf{T} \times \mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa \mathbf{N} \quad \frac{d\mathbf{B}}{ds} = -\tau \mathbf{N} \quad \kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\vec{v} \times \mathbf{a}|}{|\vec{v}|^3} = \frac{|f''(x)|}{\left[1 + (f'(x))^2\right]^{3/2}} = \frac{|\dot{y} \ddot{x} - \ddot{y} \dot{x}|}{(x^2 + y^2)^{3/2}} \quad \tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N}$$

$$\mathbf{a} = a_N \mathbf{N} + a_T \mathbf{T} \quad \text{where} \quad a_T = \frac{d|\vec{v}|}{dt} \quad \text{and} \quad a_N = \kappa |\vec{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

Fond memories from simpler times

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \quad \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Fond memories from Calculus II

$$\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C \quad \int \sqrt{a^2 + x^2} \, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + C$$

$$\int x^2 \sqrt{a^2 + x^2} \, dx = \frac{x}{8} (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2 + x^2}) + C$$