INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your name, (2) 1360/FINAL, part 1, (3) instructor’s name/class time and (4) Summer 2013 on the front of your bluebook. Also make a grading table with room for 6 problems and a total score. Work all problems. Start each problem on a new page. [Box] your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. SHOW ALL WORK

1. (a) (6 pts) Find the $x$-coordinate of the center of mass of the infinite region bounded by $x = 0, y = 0$ and $y = \lambda e^{-\lambda x}$, where $\lambda > 0$. You may use the fact that $\int_{0}^{\infty} \lambda e^{-\lambda x} dx = 1$, for $\lambda > 0$.

(b) (4 pts) Write out the full partial fraction decomposition of $\frac{3x^2 + 1}{x^4 + 3x^3 + 2x^2}$. Do not determine the coefficients.

2. (a) (4 pts) Suppose the $n^{th}$ partial sum of the series $\sum a_n$ is given by $S_n = n^{1/n}$. Find an expression for the $n^{th}$ term of the series, $a_n$. Do not simplify your answer.

(b) (4 pts) Determine whether the series above is convergent or divergent.

(c) (6 pts) Is the series $\sum_{n=100}^{\infty} \sin n \frac{\sin n}{(n^2 + \cos^2 n) \arctan n}$ absolutely convergent, conditionally convergent or divergent? Name any tests you use.

3. (a) (4 pts) What function is represented by the power series $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$? What is the radius of convergence?

(b) (4 pts) Find the sum: $\sum_{n=0}^{\infty} \frac{(-1)^n(\pi/2)^{2n+1}}{(2n+1)!}$

4. (10 pts) Consider the finite region in the first quadrant bounded by $y = x$ and $y = x^2$.

(a) Set up but do not evaluate integral(s) to find the volume of the solid formed by rotating this region about the line $x = 1$ using cylindrical shells.

(b) Same as (a), but using disks/washers.

(c) CHECK to make sure you have the proper method on the proper part above. You will lose points if you have them mixed up!

5. (8 pts) Consider the curve $f(x) = \cosh x$ between $x = 0$ and $x = 1$.

(a) Find the length of this curve.

(b) Set up but do not evaluate an integral for the surface area of the solid formed by rotating the region bounded by this curve, the $x$-axis, $x = 0$ and $x = 1$ about the $x$-axis.

6. (1 pt Extra Credit) Always true or False? If you derive the function $f(x) = \arctan x$, you get $\frac{1}{1+x^2}$.

FORMULAS ON THE NEXT PAGE.
Some identities
\[
\cos(2x) = \cos^2(x) - \sin^2(x)
\]
\[
\sin(2x) = 2\sin(x) \cos(x)
\]
\[
2\cos^2(x) = 1 + \cos(2x)
\]
\[
2\sin^2(x) = 1 - \cos(2x)
\]
\[
\cosh^2(x) - \sinh^2(x) = 1
\]
\[
2\cosh^2(x) = \cosh(2x) + 1
\]
\[
2\sinh^2(x) = \cosh(2x) - 1
\]

Moments, Mass and Center of Mass of a thin metal plate with density \( \rho \)
Mass: \( M = \text{density} \times \text{area} = \int \rho \cdot dA = \int dm \)
Moments: \( M_x = \int \hat{y} \ dm, M_y = \int \hat{x} \ dm \)
Center of Mass: \( \bar{x} = \frac{M_y}{M}, \bar{y} = \frac{M_x}{M} \)

Volume of a Solid of Revolution
Disk Method: \( \Delta V = \pi R^2 \Delta h \)
Washer Method: \( \Delta V = \pi [R^2 - r^2] \Delta h \)
Shell Method: \( \Delta V = 2\pi rh \Delta r \)

Some useful limits
\[
\lim_{n \to \infty} \frac{\ln n}{n} = 0 \quad \lim_{n \to \infty} \frac{n}{\sqrt{n}} = 1
\]
\[
\lim_{n \to \infty} x^{1/n} = 1, \quad x > 0 \quad \lim_{n \to \infty} x^n = 0, \quad |x| < 1
\]
\[
\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n = e^x, \quad \text{any } x \quad \lim_{n \to \infty} \frac{x^n}{n!} = 0, \quad \text{any } x
\]

Frequently used Maclaurin series
\[
\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1
\]
\[
e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty
\]
\[
\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad |x| < \infty
\]
\[
\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty
\]
\[
\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \leq 1
\]
\[
(1 + x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n, \quad |x| < 1
\]