1. (26 pts) Evaluate the following integrals.
   (a) $\int \frac{x^2 - x + 6}{x^3 + 3x} \, dx$
   (b) $\int (x^2 - 1)^{-3/2} \, dx$

2. (18 pts) For this problem let $I = \int_{\pi/3}^{0} \sin(x) \cos(4x) \, dx$.
   (a) Approximate $I$ using the trapezoidal approximation $T_2$. Be sure to simplify your answer.
   (b) Use the formula for $|E_T|$ to find an error bound for the approximation in part (a). You do not need to simplify your final answer.

3. (a) (10 pts) Determine whether $\int_{0}^{\infty} \frac{dx}{e^x + e^{-x} + x}$ is convergent or divergent.
   (b) (12 pts) Suppose functions $f$ and $g$ are continuous for all real numbers and that $\int_{-1}^{\infty} f(x) \, dx$ diverges and $\int_{-1}^{\infty} g(x) \, dx$ converges. No justification is necessary for the following questions about $f$ and $g$. If the answer cannot be determined, write Indeterminate. If there are no values that satisfy the conditions, write None.
   i. Find all real values of $a$ for which $\int_{-1}^{\infty} a \cdot f(x) \, dx$ diverges.
   ii. Find all real values of $b$ for which $\int_{-1}^{\infty} (b + g(x)) \, dx$ converges.
   iii. Find all real values of $c$ for which $\int_{-1}^{\infty} (f(x) + g(x)) \, dx$ converges.

4. (16 pts) Find the area of the region bounded by $y = x + \sin^{-1} x$ and the $x$-axis on the interval $[0, 1]$. 

TURN OVER—More problems on the back!
5. (18 pts) Consider the region in the first quadrant bounded by \( y = 3\sqrt{x} - 1 \) and \( y = 3\sqrt{x} - 2 \) on the interval \([1, 6]\).

(a) Sketch the curves and shade the region. Label all intercepts.
(b) Find the volume of the solid generated when the region is rotated about the \( x \)-axis.

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**Trigonometric identities**
\[
\begin{align*}
2 \cos^2(x) &= 1 + \cos(2x) \\
2 \sin^2(x) &= 1 - \cos(2x) \\
\sin(2x) &= 2 \sin(x) \cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x)
\end{align*}
\]

**Inverse Trigonometric Integral Identities**
\[
\begin{align*}
\int \frac{du}{\sqrt{a^2 - u^2}} &= \sin^{-1}(u/a) + C, \ u^2 < a^2 \\
\int \frac{du}{a^2 + u^2} &= \frac{1}{a} \tan^{-1}(u/a) + C \\
\int \frac{du}{u\sqrt{u^2 - a^2}} &= \frac{1}{a} \sec^{-1}|u/a| + C, \ u^2 > a^2
\end{align*}
\]

**Midpoint Rule**
\[
\int_{a}^{b} f(x) \, dx \approx M_n = \Delta x \left[ f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n) \right] \quad \text{where} \quad \Delta x = \frac{b - a}{n} \quad \text{and} \quad \bar{x}_i = \frac{x_{i-1} + x_i}{2}
\]
\[
|E_M| \leq \frac{K(b - a)^3}{24n^2}.
\]

**Trapezoidal Rule**
\[
\int_{a}^{b} f(x) \, dx \approx T_n = \frac{\Delta x}{2} \left[ f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n) \right] \quad \text{where} \quad \Delta x = \frac{b - a}{n} \quad \text{and}
\]
\[
|E_T| \leq \frac{K(b - a)^3}{12n^2}.
\]