APPM 1360 Exam 3 Spring 2017

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number, and instructor. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

1. (a) (14 pts) Do the following series converge absolutely, converge conditionally, or diverge? Justify your answers and name any tests you use.
   
   i. \( \sum_{n=2}^{\infty} \left( \frac{\ln(n)}{\ln(n^3)} \right)^n \)
   
   ii. \( \sum_{n=1}^{\infty} \tan(1/n) \)

   (b) (14 pts) Suppose you know that the series \( \sum_{n=1}^{\infty} \frac{2(x - 7)^n}{\sqrt{5n - 2}} \) has a radius of convergence of \( R = 1 \).

   Use this information to help determine the values of \( x \) for which the series is absolutely or conditionally convergent.

2. (17 pts)

   (a) Find a power series representation for \( \frac{1}{1 + 4x} \). Express your answer in sigma notation.

   (b) Find a power series representation for \( \frac{1}{(1 + 4x)^2} \). Express your answer in sigma notation.

   (c) What is the sum of the series

   \[ -2 \left( \frac{4}{5} \right) + 3 \left( \frac{4}{5} \right)^2 - 4 \left( \frac{4}{5} \right)^3 + \cdots + (-1)^n(n + 1) \left( \frac{4}{5} \right)^n + \cdots ? \]

3. (18 pts) Consider the function \( g \) with \( g(0) = 2 \) and \( g^{(n)}(x) = \frac{(-1)^n 2^n n!}{(1 + 2x)^{n+1}} \) for \( n = 1, 2, 3, \ldots \)

   (a) Write the formula for the Taylor series of a general function \( f(x) \) centered at \( a \). Assume that \( f \) has derivatives of all orders.

   (b) Find \( T_2(x) \), the 2nd degree Taylor polynomial of \( g \), centered at 0, and use it to approximate the value of \( g(0.1) \).

   (c) Use Taylor’s Formula to find an error bound for the approximation.

   **TURN OVER—More problems on the back!**
4. (17 pts) The binomial series for \( \frac{1}{\sqrt{1 - 3x}} \) is 

\[ 1 + \sum_{n=1}^{\infty} c_n x^n \]

where the coefficient \( c_n = \frac{1 \cdot 6 \cdot 11 \cdots (5n - 4) \cdot 3^n}{5^n n!} \) for \( n \geq 1 \).

(a) Find \( c_2 \) and \( \left(-\frac{1}{5}\right)^2\). Simplify your answers.

(b) Express the coefficient \( c_n \) in terms of \( \left(-\frac{1}{5}\right)^n \) for \( n \geq 1 \).

(c) Use the Ratio Test to find the radius of convergence of the series. (It is not necessary to find the interval of convergence.)

5. The following problems are not related.

(a) (10 pts) Use a Maclaurin series to find a power series representation for

\[ \int \ln \left(1 + \frac{x^2}{2}\right) \, dx. \]

Express your answer in sigma notation.

(b) (10 pts)

i. Use multiplication of series to find the first three nonzero terms of the Maclaurin series for

\[ \cos x \arctan x. \]

ii. Use your previous answer to evaluate

\[ \lim_{x \to 0} \frac{x^3}{\cos x \arctan x - x}. \]

Frequently Used Maclaurin Series

\[ \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \quad R = 1 \]

\[ e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad R = \infty \]

\[ \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n + 1)!} \quad R = \infty \]

\[ \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \quad R = \infty \]

\[ \tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n + 1} \quad R = 1 \]

\[ \ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} \quad R = 1 \]

\[ (1 + x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n \quad R = 1 \]