1. (a) (10 points) Find the volume of the solid obtained by rotating the region enclosed by \( x = 4y - y^2 \) and \( x = 3 \) about the line \( x = 1 \).

(b) (7 points) Set up but do not evaluate the integral that represents the volume of the solid obtained by rotating the region enclosed by \( y = \frac{3}{x+3} \) and \( y = 1 - x \) about the \( y \)-axis.

(c) (10 points) Solve the initial value problem: \( \frac{dy}{dt} = e^{2t} - y \), \( y(0) = \ln 3 \).

2. (a) (10 points) Find the arc length of \( y = (3x)^{2/3} \) on \( 0 \leq x \leq 8/3 \). Show all of your work. Give an exact answer.

(b) (8 points) Suppose the curve given in part (a) is rotated about the \( x \)-axis. Set up but do not evaluate the integral to find the surface area of the resulting solid. [Hint: Make sure your integral is not an improper integral.]

(c) (10 points) Suppose that a thin metal plate of uniform density is defined by the region bounded by \( y = 2 - x^2 \), \( y = x \), and \( x = 0 \) in the first quadrant. Further, suppose that \( M_x = 19 \). Find \( M_y \).

3. (a) (8 points) Determine whether \( \left\{ e^{-\sqrt{n}} \cos(n^2 + 1) \right\}_{n=1}^{\infty} \) converges or diverges. If it converges, find what real number it converges to. If it diverges, explain why.

(b) (3 points each) Consider the sequence \( b_n = \left\{ \frac{2^n}{n!} \right\}_{n=2}^{\infty} \).

i. Write out the first four terms of this sequence. Simplify your answers.

ii. Is the given sequence monotonic? Briefly justify your answer.

iii. Is the given sequence bounded? Briefly justify your answer.

iv. Based only on your answers to parts ii. and iii., is the given sequence convergent?

4. (a) (5 points each) Determine whether the following series converge or diverge. Justify your answers and cite any tests that you use.

i. \( \sum_{n=0}^{\infty} \frac{(-3)^{2-n}}{4^n} \)  

ii. \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{e^{1/n}} \)
(b) (7 points) Find a simple formula of the partial sum of the following series, and use it to determine whether the series converges or not.

\[ \sum_{n=2}^{\infty} \log \left( \frac{n}{n-1} \right) \]

(c) (4 points each) Consider three sequences \( \{a_n\}, \{b_n\}, \) and \( \{c_n\} \) such that \( a_n \leq c_n \leq b_n \) for all \( n \geq 1 \). Suppose that we already know

\[ \sum_{n=1}^{\infty} a_n = -10 \quad \text{and} \quad \sum_{n=1}^{\infty} b_n = 250. \]

Let \( s_n = a_1 + a_2 + \ldots + a_n \) be the partial sum of the series \( \sum_{n=1}^{\infty} a_n \). Do the following sequences or series converge? If so, find the value it converges to; if not, briefly justify why.

i. \( \lim_{n \to \infty} (3a_n + 7b_n - c_n s_n) \)

ii. \( \sum_{n=1}^{\infty} (b_n - 250) \)

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Some Trigonometric identities

\[ 2 \cos^2(x) = 1 + \cos(2x) \]
\[ 2 \sin^2(x) = 1 - \cos(2x) \]
\[ \sin(2x) = 2 \sin(x) \cos(x) \]
\[ \cos(2x) = \cos^2(x) - \sin^2(x) \]

Inverse Trigonometric Integral Identities

\[ \int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C, \quad u^2 < a^2 \]
\[ \int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1}(u/a) + C \]
\[ \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}|u/a| + C, \quad u^2 > a^2 \]

Center of Mass Integrals

\[ M = \int_a^b \rho(f(x) - g(x)) \, dx \]
\[ M_y = \int_a^b \rho x(f(x) - g(x)) \, dx \]
\[ M_x = \int_a^b \frac{1}{2} \rho(f^2(x) - g^2(x)) \, dx \]
\[ \bar{x} = \frac{M_y}{M} \quad \text{and} \quad \bar{y} = \frac{M_x}{M} \]