
On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number, and instructor. This exam is worth 100 points and has 4 questions on both sides of this paper.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

The following formulas may be of use:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$$

1. (28 points, 7 points each) Evaluate the following integrals using any method we've discussed in class.

a.) $\int_{-1/2}^{1/2} \sqrt{\frac{1}{4} - x^2} dx$

b.) $\int x(\sqrt{x} + \sqrt[3]{x}) dx$

c.) $\int_0^{\pi/4} \sec(x) \tan(x) dx$

d.) If $\int_1^5 f(x) dx = 2$, $\int_4^5 3f(x) dx = 9$, evaluate $\int_1^4 f(x) dx$.

Problems #2,3,4 continued on back side!

2. (15 points) If $10,800 \text{ cm}^2$ of material is available to make a box with a square base and an open top, find the length of the base of the box that maximizes the volume of the box. Be sure to include all units in your final answer.

3. (28 points) Consider the function $f(x) = \begin{cases} -|x + 1/2| & x < 0 \\ 1 - x^2 & x \geq 0 \end{cases}$

a.) (5 points) Sketch $f(x)$ on the interval $[-2, 1]$.

b.) (5 points) Calculate $\int_{-2}^0 f(x)dx$ using any method we've discussed in class.

c.) (10 points) Give the expression for the right endpoint sum for a given integer n , R_n , for $f(x)$ on the interval $[0, 1]$.

d.) (5 points) Use part (c) to calculate $\int_0^1 f(x)dx$

e.) (3 points) Is $f(x)$ integrable on $[-2, 1]$? Why or why not? If so, calculate $\int_{-2}^1 f(x)dx$

4. (29 points) The following are unrelated:

a.) (12 points) Let $f(x) = \sin(3x)$. Suppose Newton's Method is used to find the root(s) of $f(x)$.

i.) Plot $f(x)$ on $[0, 4\pi/3]$ and label all x and y intercepts. Sketch the tangent line at $x = \pi/6$. If we use an initial approximation of $x_1 = \pi/6$, what would our second approximation, x_2 , be?

ii.) Use Newton's Method to find an expression for x_{n+1} in terms of x_n . If $x_1 = \pi/4$, find x_2 .

b.) (12 points) A man standing on a 32 foot cliff throws a rock up in the air with vertical velocity $v(t) = -32t + 16$ feet/second. Find $s(t)$ and calculate the displacement and total distance traveled by the rock before it hits the ground. Be sure to include all units.

c.) (5 points) Write the following expression in Sigma notation:

$$4 + 7 + 10 + 13 + 16 + 19 + 22$$
