

INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1350/Final**, (3) **lecture number/instructor name** and (4) **SPRING 2018** on the front of your bluebook. Make a **grading table** for 5 problems and a total. Do all problems. **Start each problem on a new page.** Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

1. (a)(13pts) (i) What is the domain of $g(x) = \frac{x^2 + x}{x^2 + 3x + 2}$? Give your answer in interval notation. (ii) Find all horizontal asymptotes of $g(x)$, justify your answer with limits.

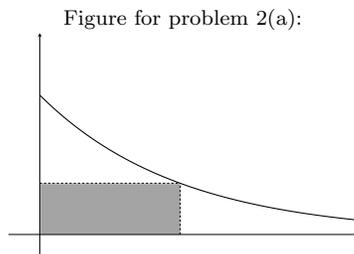
(b)(13pts) Find the real number a so that the function $f(x) = \begin{cases} \ln(x + e^{x+2}), & \text{if } x > 0 \\ a \cosh(x), & \text{if } x \leq 0 \end{cases}$ is continuous for all real numbers.

Use limits to answer this question.

- (c)(4pts) If $y = (\tan^{-1}(x))^2$ then dy/dx is equal to which of the options below? (**No justification necessary** - Choose only one answer, copy down the entire answer.)

(A) $-2 \tan^{-3}(x) \sec^2(x)$ (B) $\frac{2}{1+x^2}$ (C) $\frac{2 \tan^{-1}(x)}{1+x^2}$ (D) $2 \arctan(x) \operatorname{arcsec}^2(x)$

2. (a)(15pts) What is the area of the largest rectangle in the first quadrant with two sides on the axes and one corner on the curve $y = e^{-x}$? Show all work and be sure to classify your answer either using the First Derivative Test or the Second Derivative Test.



- (b)(15pts) Evaluate the definite integral $\int_1^{\sqrt{3}} \frac{6}{1+x^2} dx$. Simplify your answer.

- (c)(5pts) If $f(x) = \frac{\operatorname{sech}^2(x)}{2 + \tanh(x)}$, then which of the choices below corresponds to $\int f(x) dx$? (**No justification necessary** - Choose only one answer, copy down the entire answer.)

(A) $\frac{(2 + \tanh(x))^2}{2} + C$ (B) $\ln(\tanh(x)) + C$ (C) $\frac{2 \operatorname{sech}(x) \tanh(x)}{2 + \tanh(x)} + C$ (D) $\ln|2 + \tanh(x)| + C$

3. (a)(13pts) Use logarithmic differentiation to find the derivative of $y = (\sec x)^{\ln x}$. Show all work.

- (b)(13pts) Find the linearization of $f(x) = \int_0^{\sin(x)} \sqrt{1+t^2} dt$ centered at the point $a = \pi$. Show all work.

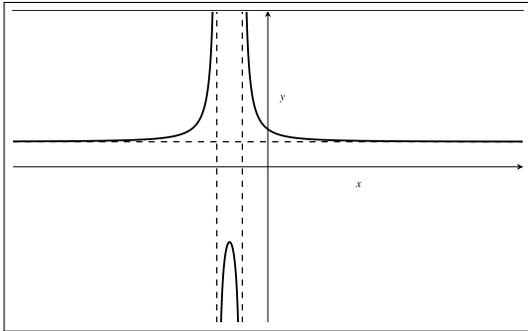
- (c)(4pts) The definite integral $\int_0^1 x e^{-2x^2} dx$ is equal to which choice below? (**No justification necessary** - Choose only one answer, copy down the entire answer.)

(A) $\frac{1}{4}(1 - e)$ (B) $\frac{1}{4}(e^{-2} - 1)$ (C) $\frac{1 - e^{-2}}{4}$ (D) $-e^{-1} - e$

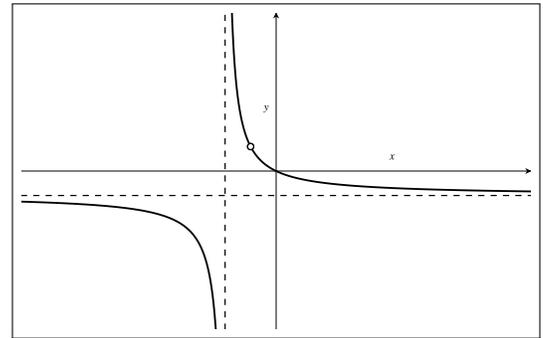
4. (a)(15pts) Use l'Hospital's Rule to evaluate the limit $\lim_{x \rightarrow \infty} x \tan(1/x)$. Show all work.

(b)(15pts) The *half-life* of Cesium-137 is 30 years. Suppose we initially have a 100-mg sample. (i) Find a formula for the mass remaining after t years. (ii) Set-up (**but do not evaluate**) an integral to calculate the *average value* of the mass remaining of Cesium-137 after 10 years.

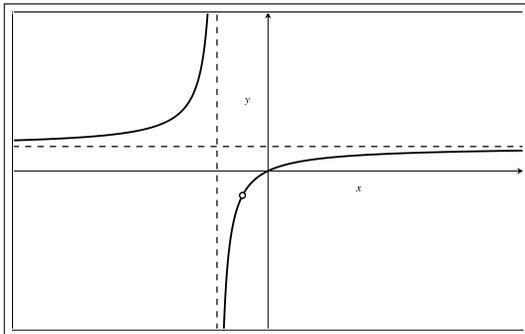
(c)(5pts) Which graph below most closely resembles the graph of $g(x) = \frac{x^2 + x}{x^2 + 3x + 2}$? (**No justification necessary** - Choose only one answer.)



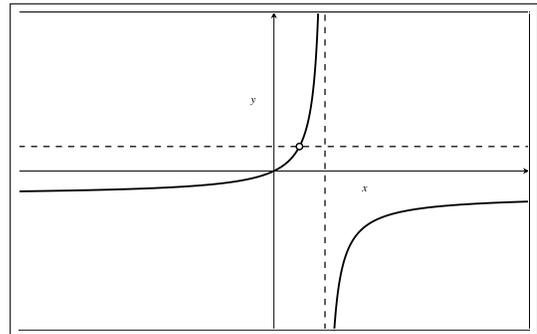
(A)



(B)



(C)



(D)

5. (20pts) Answer either **ALWAYS TRUE** or **FALSE**. You do NOT need to justify your answer. (*Don't just write down "A.T." or "F", completely write out the words "ALWAYS TRUE" or "FALSE" depending on your answer.*)

(a)(5 pts) If the velocity of a particle at time t seconds is $v(t) = 2t - 1$ meters per second, then the *total distance* traveled during the time period $0 \leq t \leq 1$ by the particle is 0.25 meters.

(b)(5 pts) By the Intermediate Value Theorem, the equation $\log_2(x) + x = 0$, for $0.5 \leq x \leq 4$, has at least one root in the interval $(0.5, 4)$.

(c)(5 pts) If the function $f(x)$ is continuous for all real values of x then $f(x)$ is differentiable for all real values of x .

(d)(5 pts) If $f(x) = \ln(x) + \tan^{-1}(x)$ then $f(1) = \pi/4$ and $(f^{-1})'(\pi/4) = 2/3$.