INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1350/Final**, (3) **lecture number/instructor name** and (4) **SPRING 2017** on the front of your bluebook. Make a **grading table** for 5 problems and a total. Do all problems. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

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1. (a)(13pts) Use the Squeeze Theorem to evaluate the limit: $\lim_{x\to 0} 2|x|\sin(1/x^2)$. Show all work.

   (b)(13pts) Evaluate the limit $\lim_{x\to\infty}(e^x + x)^{1/x}$. Show all work, justify your answer.

   (c)(6pts) For this problem choose only one choice from below, **no justification necessary**- *Copy down the entire answer*. Suppose $g(x) = \dfrac{(x-2)\ln(x)}{x^2+x-6}$, then $g(x)$ has a *vertical asymptote* at:

   (A) $x=-3, x=2$ (B) $x=-3$ (C) $x=-3,\ x=0$ (D) $x=0$ (E) $x=0,\ x=2$

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2. (a)(13pts) Consider the function $f(x)=\begin{cases}\sinh\left(\pi e^x - \cos^{-1}(x-1)\right), & \text{if } x\le 0\\ x\ln(x), & \text{if } x>0\end{cases}$.

   (i) Evaluate the limit $\lim_{x\to 0^+} f(x)$. Show all work.

   (ii) Is the function $f(x)$ continuous at $x=0$? Justify your answer with a limit. (If $f(x)$ is not continuous at $x=0$ then what type of discontinuity is present at $x=0$? Explain.)

   (b)(13pts) Julien is jogging around a circular track of radius 50m. In a coordinate system with origin at the center of the track, Julien's x-coordinate is changing at a rate of -1.25 m/s when their coordinate is $(40, 30)$. Find dy/dt at this moment. (Recall that the equation of a circle with radius r is $x^2+y^2=r^2$.)

   (c)(6pts) In your bluebook, clearly sketch the graph of $g(x) = 3x^{1/3} - x = x^{1/3}(3-x^{2/3})$, note that $g'(x) = \dfrac{1}{x^{2/3}} - 1$ and $g''(x) = -\dfrac{2}{3x^{5/3}}$. Label all intercepts, extrema, inflection points and asymptotes. (Recall that $\sqrt{3}\approx 1.7$)

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3. (a)(13pts) Evaluate the integral: $\displaystyle\int_0^{\cosh^{-1}(\sqrt{3})} \frac{\sinh(x)}{1+(\cosh(x))^2}\,dx$. Show all work.

   (b)(13pts) Find the area between the positive function $f(x)=e^x\sqrt{e^x+1}$ and the x-axis from $x=\ln(3)$ to $x=\ln(8)$.

   (c)(7pts) Which of the five choices given below is the limit $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\sin^{-1}\left(\frac{i}{n}\right)\frac{i}{n^2}$ equivalent to? Choose only one answer, **no justification necessary** - *be sure to copy down the entire answer*.

   (A) $\displaystyle\int_0^1 \sin^{-1}(x)\,dx$ (B) $\displaystyle\int_0^1 \frac{x}{\sin(x)}\,dx$ (C) $\displaystyle\int_0^1 x\sin(x/n)\,dx$ (D) $\displaystyle\int_0^1 x\sin^{-1}(x)\,dx$ (E) $\displaystyle\int_{-1}^0 x\sin^{-1}(x)\,dx$

PROBLEMS #4 & #5 ON THE OTHER SIDE

4. (a)(13pts) Use logarithmic differentiation to find dy/dx: $(i)\ y = \dfrac{x\sin(x)}{\sqrt{\sec(x)}}$ $(ii)\ y = (\tan^{-1}(x))^x$

(b)(13pts) A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420. (i) Find the relative growth rate of the population. (ii) Find the number of bacteria after 3 hours.

(c)(7pts) Which of the five choices given below is the antiderivative $\displaystyle\int \frac{5^x}{1+5^x}\,dx$ equivalent to? (Choose only one answer, **no justification necessary** - *be sure to copy down the entire answer*.)

(A) $\tan^{-1}(5^{x/2}) + C$ (B) $\dfrac{1}{(1+5^x)^2} + C$ (C) $\ln\left(\dfrac{1+5^x}{5}\right) + C$ (D) $\ln(1+5^x) - \ln(5) + C$ (E) $\dfrac{\ln(1+5^x)}{\ln(5)} + C$

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5. (20pts) Answer either **ALWAYS TRUE** or **FALSE**. You do NOT need to justify your answer. (*Don't just write down "A.T." or "F", completely write out the words "ALWAYS TRUE" or "FALSE" depending on your answer.*)

(a)(5 pts) Suppose $f(x)$ is continuous for all x, then $f(x)$ is differentiable at $x = a$ if $\displaystyle\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists.

(b)(5 pts) The linearization of $g(x) = \ln(x+e)$ centered at $a = e$ is $L(x) = \ln(2) + \dfrac{x}{2e} - \dfrac{1}{2}$.

(c)(5 pts) The Mean Value Theorem applies to $h(x) = -1/x$ on $[-3, -1/2]$ and the value of c that satisfies the Mean Value Theorem is $c = -2/3$.

(d)(5 pts) If $f(x) = \displaystyle\int_2^x \sqrt{1+3^t}\,dt$, then $f(2) = 0$ so $f^{-1}(0) = 2$ and $(f^{-1})'(0) = \dfrac{1}{\sqrt{10}}$.

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