INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1350/Exam 3**, (3) **lecture number/instructor name** and (4) **SPRING 2017** on the front of your bluebook. Make a **grading table** for 4 problems and a total. Do all problems. **Start each problem on a new page. Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

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1. (a)(12pts) Suppose we are trying to find the dimensions of a rectangle with area 1000 m^2 whose perimeter is to be as small as possible using optimization. Suppose x and y represent the dimensions of the rectangle. Answer the following questions:

   (i)(3pts) Is this is a *minimization* or *maximization* problem? Write down a function in terms of x and y that you would minimize (or maximize). Your answer should be in terms of the variables x and y only.

   (ii)(3pts) Write down an equation that relates the variables x and y.

   (iii)(6pts) Now using optimization find the value of x and y that satisfy this problem. Justify your answer by classifying your critical point(s) using either the 1st or 2nd Derivative Test.

   (b)(12pts) Suppose we want to approximate the intersection points of $f(x) = x^2$ and $g(x) = \cos(x)$. If we want to use Newton's Method to do this then what would the formula for x_{n+1} be? (To get full points for this question you must provide the explicit formula for x_{n+1} in terms of x_n, the generic formula for Newton's Method is not sufficient. You do **not** need to approximate the point of intersection.)

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2. (a)(12pts) Approximate the area of the region bounded by the function $f(x) = 2 + \sin(x)$, $0 \leq x \leq \pi$ and the x-axis using four approximating rectangles and taking the sample points to be the right endpoints.

   (b)(12pts) Use the formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

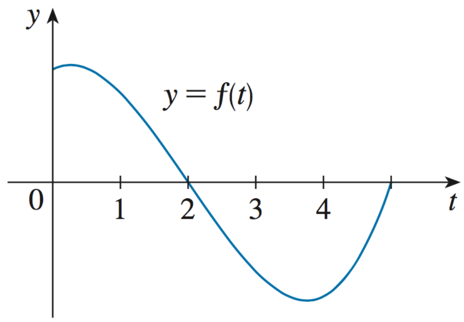
   to evaluate the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\frac{i}{n} + \frac{i^2}{n^2}\right]$. Show all work.

PROBLEMS #3 & #4 ON THE OTHER SIDE

3. (a)(12pts) Use a definite integral to find the net area of the region bounded by the function $g(x) = x\sec(x^2-1)\tan(x^2-1)$ and the x-axis between $x=1$ and $x=\sqrt{2}$.

(b)(8pts) If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t, what does $\int_0^{120} r(t)\,dt$ represent? Explain your answer. (Write your answer in complete sentences.)

(c)(6pts) If $F(x) = \int_2^x f(t)\,dt$, where f is the function whose graph is given below, put the following values in order from smallest to largest: $F(0)$, $F(2)$, $F(3)$. (No justification necessary, box your answer.)



Graph for Problem 3(c).

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4. (a)(10pts) Evaluate the indefinite integral $\int x\sqrt{x-1}dx$. Show all work.

(b)(10pts) Find the derivative of the function $f(x) = \int_{1/x}^{2} \sin^2(t)\,dt$. Justify your answer.

(c)(6pts) Suppose the average value of $h(x)$ on the interval $[1,4]$ is $h_{\text{ave}} = \pi$, which of the following choices below is true? (**No justification necessary** - *Copy down the entire answer*)

(A) $\int_5^{20} h(x/5)dx = 3\pi/5$ (B) $\int_5^{20} h(x/5)dx = 2\pi$ (C) $\int_5^{20} h(x/5)dx = 15\pi$ (D) $\int_5^{20} h(x/5)dx = 8\pi$

— **END** —