

**INSTRUCTIONS:** Books, notes, and electronic devices are not permitted. Write (1) **your full name**, (2) **1350/Exam 2**, (3) lecture number/instructor name and (4) **SPRING 2017** on the front of your blue-book. Make a **grading table** for 4 problems and a total. Do all problems. **Start each problem on a new page.** **Box** your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

1. (a)(10pts) Find  $y'$  if  $y = \tan(x - \cos(x^2))$ .

(b)(10pts) If  $xg(x) + 2g'(x) = x^2$  and  $g'(0) = \pi$  and  $g(0) = -10$ , find  $g''(0)$ .

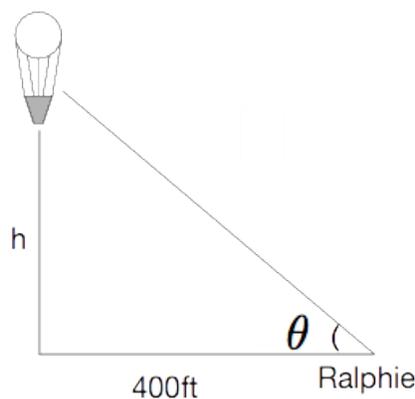
(c)(5pts) Which of the five choices given below is equivalent to  $y'$  if  $y = \left[ \frac{\sin x}{x+1} \right]^2$ ? Pick only one answer, **no justification necessary** - be sure to copy down the entire answer, don't just write down the letter of your choice:

$$(A) \frac{2 \sin(x) \cos(x)}{2(x+1)} \quad (B) \frac{\cos(x)(x+1) - \sin(x)}{(x+1)^2} \quad (C) \frac{2 \sin(x) \cos(x)}{(x+1)^2} - \frac{2 \sin^2(x)}{(x+1)^3}$$

$$(D) \frac{\cos^2(x)}{(x+1)^2} \quad (E) \frac{\cos^2(x)(x+1) - \sin(x)}{(x+1)^4}$$

2. (a)(10pts) Suppose that  $3 \leq f'(x) \leq 5$  for all values of  $x$ . Show that  $18 \leq f(8) - f(2) \leq 30$ . *Hint:* This problem can be done using one of the theorems we studied. Be sure to state the theorem you are using and justify why it applies. Show all work.

(b)(15pts) Ralphie is observing balloon launches with a telescope. Suppose a balloon is rising 400 feet from where Ralphie is standing. If the balloon rises at a rate of 20 feet per minute, how fast is the angle between the ground and the telescope changing when the balloon is 300 feet high? (See diagram below.)



Problem 2: Ralphie observes a balloon launch!

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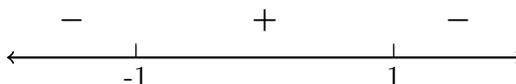
3. (a)(10pts) Find the absolute minimum and maximum values of  $f(x) = x\sqrt{4-x^2}$  for  $-1 \leq x \leq 2$ . (Be sure to write down the  $x$ -coordinate and the  $y$ -coordinate of all absolute extrema.)

(b)(10pts) Find the local maximum and minimum values of  $g(x) = \frac{x}{x^2+1}$  if  $g'(x) = \frac{1-x^2}{(x^2+1)^2}$ . Justify your answer with a derivative test. (Be sure to write down the  $x$ -coordinate and the  $y$ -coordinate of all local extrema.)

(c)(5pts) In your blue book clearly sketch the graph of a function  $h(x)$  that satisfies all the following properties (label all extrema, inflection points and asymptotes):

- $h(x)$  is a polynomial and an odd function,  $h(0) = 0$  and  $h(1) = 2$ ,
- $\lim_{x \rightarrow -0.5^+} h(x) = -1.5$ ,  $\lim_{x \rightarrow -\infty} h(x) = +\infty$ , and  $\lim_{x \rightarrow \infty} h(x) = -\infty$ ,
- The 1st derivative test for  $h(x)$  is shown below:

Sign chart for  $h'(x)$



- $h''(x) < 0$  on  $(-1/2, 0)$  and  $(1/2, \infty)$ .

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4. (a)(15 pts) The position function of a particle is given by  $s(t) = t^3 - 3t^2$  where  $t$  is in seconds and distance is in feet. (i) When is the particle at rest? (ii) What is the *total distance* traveled by the particle in the first 4 seconds?

(b)(10 pts) Use differentials to estimate the allowable *percentage error* in measuring the radius  $r$  of a sphere if the volume of the sphere is to be calculated correctly to within 6% . (Note: The volume is  $V = \frac{4}{3}\pi r^3$ .)

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