APPM 1350 **Final Exam** **Fall 2017**

On the front of your bluebook, please write: a grading key, your name, student ID, lecture number, and instructor name. This exam is worth 150 points and has 8 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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1. (26 pts) Evaluate the following.
   (a) Let $g(x) = \cos^3(\pi - 2x)$. Find $g'(\pi/3)$.
   (b) Let $y = (\sqrt{x})^x$. Find $y'(4)$.
   (c) $\lim_{r\to 0^+} e^{-1/r}\ln(r)$

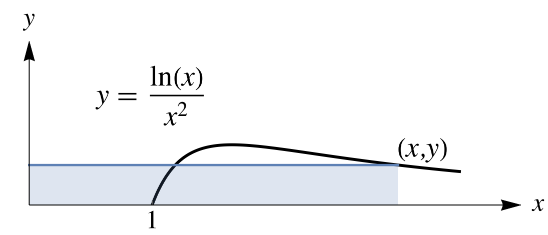
2. (24 pts) Evaluate the following integrals.
   (a) $\displaystyle\int \frac{3x^3+5x+7}{x^2}\,dx$ (b) $\displaystyle\int_{\pi^2/16}^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ (c) $\displaystyle\int (\tanh x)\ln(\cosh x)\,dx$

3. (18 pts) For each of the following unrelated problems, identify the relevant theorem, check that its hypotheses are met, and use the theorem to solve the problem.
   (a) Let $f(x) = (x-1)^2(x+2)(x+3)$. Is there a point $x = a$ such that $f(a) = 1$? Justify your answer.
   (b) If $f(1) = 12$, f' is continuous, and $\displaystyle\int_1^6 f'(x)\,dx = 18$, what is the value of $f(6)$?
   (c) Consider the curve $y = \sqrt{x} + \arccos x$ on $[0, 1/2]$. Suppose m equals the slope of the line connecting the endpoints of the curve on the given interval. Must there exist a value c in $(0, 1/2)$ such that $m = y'(c)$? Justify your answer. (*Note:* It is not necessary to calculate m.)

4. (12 pts) Let $g(x) = \sqrt{\dfrac{3e^{4x}}{e^x - 1}}$.
   (a) Does g have any vertical asymptotes? Justify your answer using appropriate limit(s).
   (b) Find $g(\ln 2)$ and simplify your answer.

TURN OVER—More problems on the back!

5. (12 pts) The rectangle shown has one side on the positive y-axis, one side on the positive x-axis, and its upper right corner on the curve $y = (\ln x)/x^2, \quad x \geq 1$. Find the maximum area of the rectangle.



6. (24 pts) The following problems are not related.

   (a) Evaluate $\displaystyle\sum_{k=21}^{100} 2k$.

   (b) Consider the function $g(x) = k \arctan x + b$ where k and b are constants.

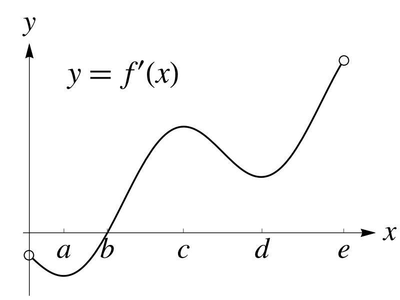
      i. Find $g'(1)$.

      ii. If the curve $y = g(x)$ is tangent to the line $y = -2x$ at $x = 1$, find the values of k and b.

   (c) Find the inverse of $y = \dfrac{2 - \tan x}{3 + \tan x}, \quad 0 < x < \pi/2$. (You may assume that the function is one-to-one.)

7. (12 pts) A dose of a painkiller is administered to a patient. Suppose the rate at which the drug is eliminated from the body is proportional to the amount present. If the painkiller has a half-life of seven hours, how long will it take for 90% of the drug to be eliminated?

8. (a) (10 pts) The graph of the first derivative f' of a function f is shown at right. Answer the following questions about the function f which is defined on $[0, e]$. (List all answers that apply. Use interval notation where appropriate. No explanation is necessary.)



      i. On what intervals is f increasing?

      ii. At what values of x does f have a local minimum value?

      iii. On what intervals is f concave up?

      iv. At what values of x does f have an inflection point?

   (b) (12 pts) Sketch a graph of a single function $y = g(x)$ with all of the following properties:

   - $\lim_{x \to a} g(x) = g(a)$ for all a except $a = 4$
   - $\lim_{x \to 4} g(x) = -\infty$
   - $g(2) = 4$
   - $g(5) = 2$
   - $\displaystyle\lim_{h \to 0} \frac{g(2+h) - g(2)}{h} = 5$
   - $\displaystyle\int_5^7 g(x)\, dx = 0$

Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$