1. (26 pts) Evaluate the following.
   (a) Let \( g(x) = \cos^3(\pi - 2x) \). Find \( g'(\pi/3) \).
   (b) Let \( y = (\sqrt{x})^x \). Find \( y'(4) \).
   (c) \( \lim_{r \to 0^+} e^{-1/r} \ln(r) \)

2. (24 pts) Evaluate the following integrals.
   (a) \( \int \frac{3x^3 + 5x + 7}{x^2} \, dx \)
   (b) \( \int_{\pi^2/16}^{\pi^2/4} \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx \)
   (c) \( \int \ln(\cosh x) \, dx \)

3. (18 pts) For each of the following unrelated problems, identify the relevant theorem, check that its hypotheses are met, and use the theorem to solve the problem.
   (a) Let \( f(x) = (x - 1)^2(x + 2)(x + 3) \). Is there a point \( x = a \) such that \( f(a) = 1 \)? Justify your answer.
   (b) If \( f(1) = 12, f' \) is continuous, and \( \int_{1}^{6} f'(x) \, dx = 18 \), what is the value of \( f(6) \)?
   (c) Consider the curve \( y = \sqrt{x} + \arccos x \) on \([0, 1/2]\). Suppose \( m \) equals the slope of the line connecting the endpoints of the curve on the given interval. Must there exist a value \( c \) in \((0, 1/2)\) such that \( m = y'(c) \)? Justify your answer. (Note: It is not necessary to calculate \( m \).)

4. (12 pts) Let \( g(x) = \sqrt{\frac{3e^{4x}}{e^x - 1}} \).
   (a) Does \( g \) have any vertical asymptotes? Justify your answer using appropriate limit(s).
   (b) Find \( g(\ln 2) \) and simplify your answer.
5. (12 pts) The rectangle shown has one side on the positive y-axis, one side on the positive x-axis, and its upper right corner on the curve \( y = \frac{\ln(x)}{x^2}, \ x \geq 1 \). Find the maximum area of the rectangle.

6. (24 pts) The following problems are not related.
   (a) Evaluate \( \sum_{k=21}^{100} 2k \).
   (b) Consider the function \( g(x) = k \arctan x + b \) where \( k \) and \( b \) are constants.
      i. Find \( g'(1) \).
      ii. If the curve \( y = g(x) \) is tangent to the line \( y = -2x \) at \( x = 1 \), find the values of \( k \) and \( b \).
   (c) Find the inverse of \( y = \frac{2 - \tan x}{3 + \tan x}, \ 0 < x < \pi/2 \). (You may assume that the function is one-to-one.)

7. (12 pts) A dose of a painkiller is administered to a patient. Suppose the rate at which the drug is eliminated from the body is proportional to the amount present. If the painkiller has a half-life of seven hours, how long will it take for 90% of the drug to be eliminated?

8. (a) (10 pts) The graph of the first derivative \( f' \) of a function \( f \) is shown at right. Answer the following questions about the function \( f \) which is defined on \([0, e]\). (List all answers that apply. Use interval notation where appropriate. No explanation is necessary.)
   i. On what intervals is \( f \) increasing?
   ii. At what values of \( x \) does \( f \) have a local minimum value?
   iii. On what intervals is \( f \) concave up?
   iv. At what values of \( x \) does \( f \) have an inflection point?
   (b) (12 pts) Sketch a graph of a single function \( y = g(x) \) with all of the following properties:
   - \( \lim_{x \to a} g(x) = g(a) \) for all \( a \) except \( a = 4 \)
   - \( g(2) = 4 \)
   - \( \lim_{h \to 0} \frac{g(2 + h) - g(2)}{h} = 5 \)
   - \( \lim_{x \to 4} g(x) = -\infty \)
   - \( g(5) = 2 \)
   - \( \int_5^7 g(x) \, dx = 0 \)

Formulas
\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2
\]