

On the front of your bluebook, please write: a grading key, your name, student ID, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

1. (30 pts) Evaluate and fully simplify all answers.

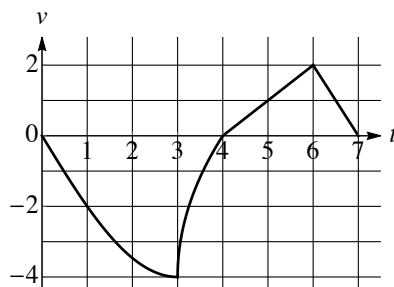
(a) $\int \frac{\sqrt[3]{1 + \sqrt{t}}}{\sqrt{t}} dt$

(b) $\int_e^{e^2} \frac{2}{x \ln(x^2)} dx$

(c) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin^3(\theta) + \cos^2(\theta) \sin(\theta)| d\theta$

(d) Consider the function $f(x) = \int_1^{x^2} \sqrt{1 + t^3} dt$. Find $f'(2)$.

2. (15 pts) The graph below shows the velocity v (in cm/sec) of an object moving up and down for $t = 0$ to 7 sec. (For parts (a) to (c) no explanation is necessary.)



- (a) On which interval(s) was the acceleration negative? Express your answer in interval notation.
- (b) At what time was the object farthest away from the starting point?
- (c) Did the object end up above, below, or at the starting position?
- (d) What was the average velocity of the object on $[5, 7]$? Justify your answer.

TURN OVER—More problems on the back!

3. (16 pts) The following problems are not related.

- (a) Express $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^5}$ as a definite integral and use it to find the value of the limit.
- (b) Evaluate the upper sum for $f(x) = (x - 1)^2$, $-3 \leq x \leq 3$, with $n = 3$ equal subintervals.

4. (24 pts) The following problems are not related.

- (a) Write the following expression in sigma notation and find the sum.

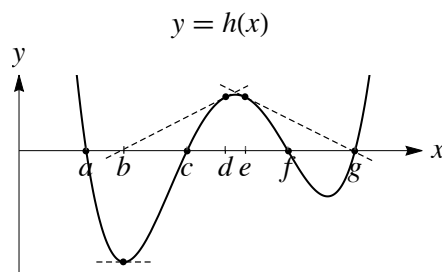
$$(1^3 + 10) + (2^3 + 20) + (3^3 + 30) + \cdots + (20^3 + 200)$$

- (b) Consider the function

$$f(x) = \frac{(1-x)(2+x)^2(3-x/2)}{(4+x)^2}.$$

Use logarithmic differentiation to find $f'(0)$.

- (c) Suppose Newton's Method is used to find a root of the function $h(x)$. The tangent lines at $x = b, d, e$ are shown. (No explanation is necessary for the following questions.)



- i. For which of the initial approximations $x_1 = a, b, c, d, e, f$, and g will Newton's Method fail to find a root? List all that apply.
- ii. Which of the initial approximations $x_1 = b, d, e$, and g will lead to a second approximation x_2 that equals one of the roots of h ? List all that apply.

5. (15 pts) Let $f(x) = \frac{5e^x}{4 - e^x}$.

- (a) Find the domain of f in interval notation.
- (b) Find the inverse function f^{-1} . (You may assume that f is one-to-one.)
- (c) If $(f^{-1})'(5/3) = 1/f'(a)$, what is the value of a ? (*Hint*: It is not necessary to differentiate the functions to find the answer.)

Formulas

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$