
On the front of your bluebook, please write: a grading key, your name, student ID, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
 - **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points.
 - Notes, papers, calculators, cell phones, and other electronic devices are not permitted.
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1. (28 pts) The following problems are not related.

(a) Let $y = \cos^4(3u^2)$. Find dy/du .

(b) Let $x\sqrt{y+4} = y^2 - 4^2$. Find dy/dx at the point $(3, 5)$.

(c) Suppose that $-2 \leq f'(x) \leq 10$ for all values of x . Find the smallest and largest possible values of $f(15) - f(6)$.

(d) Let $y = \frac{10x^3 - 6x^2 + 15x}{2x^2 + 3}$. Find an equation for the slant asymptote of y .

(It is not necessary to evaluate any limits when justifying your answer.)

2. (15 pts) Beatrice takes a 36π cm³ lump of clay and shapes it into a cylinder of length L and radius r . As she rolls the clay the cylindrical shape is maintained. How fast is the radius r changing when $L = 9$ cm and L is increasing at a rate of $\frac{1}{2}$ cm/sec?

3. (27 pts) Consider the function $y = \frac{1}{x(x-3)^2}$ with $y' = \frac{3(1-x)}{x^2(x-3)^3}$ and $y'' = \frac{6(2x^2 - 4x + 3)}{x^3(x-3)^4}$.

- Find the domain of the function in interval notation.
- Does the function have vertical asymptotes? Justify your answer using appropriate limits.
- Does the function have horizontal asymptotes? Justify your answer using appropriate limits.
- On what intervals is y increasing? decreasing?
- Find the x and y coordinates of the local maximum and minimum extrema, if any.
- On what intervals is y concave up? concave down?
- Sketch a graph of y . Clearly label any asymptotes and local extrema.

TURN OVER—More problems on the back!

4. (14 pts) Find the x and y coordinates of the absolute maximum and minimum values (if any) of

$$f(x) = \frac{3 \sin(x)}{5 + 2 \sin(x)} \text{ on } [-\pi, \pi].$$

5. (16 pts) Shown below is the tangent line to a function $y = f(x)$ at $x = 6$.

- Use the graph to find an equation for the tangent line.
- Find the values of $f(6)$ and $f'(6)$.
- Use a linear approximation to estimate the value of $f(6.02)$.
- Now suppose $f(x)$ is a cubic function $ax^3 + bx^2 + cx + d$ where $a, b, c,$ and d are constants. If the value of $f''(6)$ is 0, find the ratio a/b .

