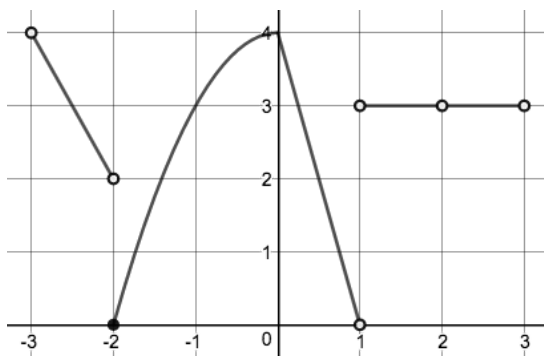


On the front of your bluebook, please write: a grading key, your name, student ID, lecture number, and instructor name. This exam is worth 100 points and has 6 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Limit problems should not be evaluated using L'Hopital's Rule. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

1. (19 pts) Using the function $f(x)$ plotted in the figure, find the following values, or write DNE if they do not exist. No justification is necessary for this problem.



- (a) $\lim_{x \rightarrow -2} f(x)$ (d) $f(1)$
- (b) $\lim_{x \rightarrow -2^-} f(x)$ (e) $\lim_{x \rightarrow 2} f(x)$
- (c) $\lim_{x \rightarrow -2^+} f(x)$ (f) $f'(\frac{3}{5})$
- (g) the average rate of change of f on $[-2, \frac{1}{2}]$
- (h) the values of x in $(-3, 3)$ where f is not differentiable
2. (10 pts) Use the graph of $f(x)$ shown above to sketch a graph of the derivative $f'(x)$. Label tick marks clearly.
3. (17 pts) The following problems are not related.

- (a) If $\tan \theta = 1/x$, $\pi < \theta < \frac{3\pi}{2}$, what is the value of $\csc \theta$ in terms of x ?
- (b) Find the domain of the function $y = \frac{x}{\sqrt{x+2}-3}$. Express your answer in interval notation.
- (c) Let $f(x) = 7 - 2x$. Suppose we use the precise definition of a limit (**see below**) to verify the value of $\lim_{x \rightarrow 1/2} (7 - 2x)$ and we find that if $0.4 < x < 0.6$ then $5.8 < f(x) < 6.2$. What are the corresponding values of ϵ and δ ?
- (d) Let $h(x) = |x^3 + 1|$.
- Sketch a graph of $y = h(x)$. Label all intercepts.
 - Express h as a piecewise-defined function without using absolute value.

TURN OVER—More problems on the back!

Precise Definition of a Limit

The limit of $f(x)$ as x approaches a is L if for every number $\epsilon > 0$ there is a corresponding number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

4. (26 pts)

(a) Does the function $y = \frac{\cos^2(x) - \sin^2(x)}{\cos(x) - \sin(x)}$ have a vertical asymptote at $x = \pi/4$? Justify your answer by evaluating appropriate limits.

(b) Does the function $y = \frac{6x + 2}{\sqrt{4x^2 - 2}}$ have horizontal asymptotes? Justify your answer by evaluating appropriate limits.

(c) Evaluate $\lim_{x \rightarrow 0} \frac{x \cot(2x)}{5}$.

5. (14 pts) Consider the function $f(x) = \begin{cases} \frac{x^2 + 2x - 3}{x - 1} & x < -1 \\ c & x = -1 \\ b \cos(\pi x) & x > -1. \end{cases}$

(a) Write the definition of continuity of an arbitrary function f at a number a .

(b) Are there constants b and c that make $f(x)$ continuous at $x = -1$? Use the definition of continuity to find b and c , or to explain why they don't exist.

6. (14 pts) Let $g(x) = \frac{1}{7 - 2x}$.

(a) Write the limit definition of the derivative of an arbitrary function $f(x)$ at a number a .

(b) Use your answer for part (a) to find the value of $g'(0)$.

(c) Find an equation for the line tangent to $y = g(x)$ at $x = 0$.