

Print Name \_\_\_\_\_

APPM 1350

Final Exam

Fall 2016

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 150 points and has 7 questions on both sides of this paper.

- Include this exam sheet in your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

1. (25 pts) Evaluate the following integrals.

(a)  $\int_0^2 \frac{4x}{(x^2 + 1)^2} dx$       (b)  $\int \frac{5^t}{5^t - 1} dt$       (c)  $\int_{-1/2}^0 \frac{6}{\sqrt{1 - x^2}} dx$

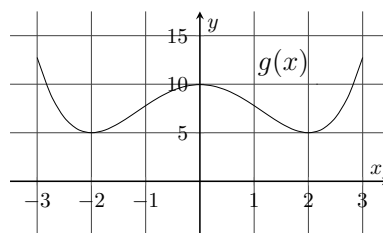
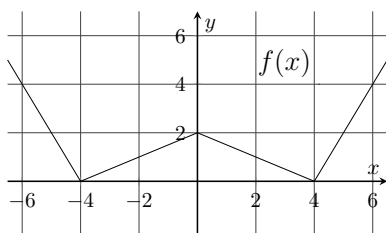
2. (26 pts) The following problems are not related.

- (a) Show that the function  $h(x) = \cos x + 2x + 1$  has at least one real root. Indicate an interval where the root can be found.
- (b) Find an equation for the line tangent to  $y = \sin^3(2x)$  at  $x = \pi/6$ . Express your answer in the form  $y = mx + b$ .
- (c) Use the graphs of  $f(x)$  and  $g(x)$ , shown below, to find the following values. (No justification is required.)

i.  $f'(-3) + f'(1)$

ii.  $f'(g(2))$

iii.  $(fg)'(2)$



TURN OVER—More problems on the back!

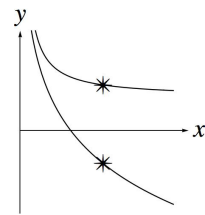
3. (18 pts)

- (a) Find the value of  $\lim_{x \rightarrow 0} \arctan\left(\frac{1}{x}\right)$ .
- (b) Use the Squeeze Theorem to find the value of  $\lim_{x \rightarrow 0} |x| \arctan\left(\frac{1}{x}\right)$ .
- (c) Does  $\lim_{x \rightarrow \infty} \frac{\arctan(1/x^2)}{\arctan(1/x)}$  exist? Justify your answer.

4. (16 pts) For this problem let  $f(x) = (\ln x)^x$ .

- (a) Find the domain of  $f$ .
- (b) Find the instantaneous rate of change of  $f$  with respect to  $x$ .
- (c) Find the value of  $\lim_{h \rightarrow 0} \frac{(\ln(e+h))^{e+h} - 1}{h}$ .

5. (20 pts) Bug A is moving along the curve  $y = 1 + 1/x$  and Bug B is moving along the curve  $y = 1 - 2 \ln x$  so that the bugs are always vertically aligned (one directly above the other).



- (a) The distance between the two bugs is minimized at what  $x$ -coordinate?
- (b) As Bug A reaches  $x = 2$ , its  $y$ -coordinate is decreasing at a rate of 0.1 unit/sec. How fast is Bug B's  $y$ -coordinate changing then?

6. (25 pts) The following problems are not related.

- (a) Find all asymptotes (if any) of the function  $h(x) = \frac{e^x}{2 - e^x}$ . Justify your answer using limits.
- (b) Let  $y = x^2 \sqrt{5 - x}$ . Find (i) the domain of the function, and (ii) the intervals of increase and decrease. Express your answers in interval notation.

7. (20 pts) The following problems are not related.

- (a) Find the sum of  $\sum_{n=1}^{10} (\ln(3n) - \ln(2n))$ . Simplify your answer.
- (b) Let  $g(x) = \int_0^x \tanh(t^2 - t - 2) dt$ . Find  $g'(x)$  and  $g''(x)$ .
- (c) A sample of radioactive cesium-137 with an initial mass  $m$  of 50 mg decays at the rate of

$$\frac{dm}{dt} = -\frac{(\ln 2)m}{30} \text{ mg/year.}$$

Find an expression for  $m(t)$ , the mass remaining after  $t$  years. Simplify your answer.

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### Formulas

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$