

Print Name \_\_\_\_\_

**APPM 1350**

**Exam 3**

**Fall 2016**

**On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor.** This exam is worth 100 points and has 5 questions on both sides of this paper.

- Include this exam sheet in your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

1. (21 pts) Evaluate each of the following.

(a)  $\int \frac{\sec^2(1/x^2)}{x^3} dx$       (b)  $\int_0^2 \frac{t}{\sqrt{1+2t^2}} dt$       (c)  $\int_2^{x^2} \frac{3}{2t+1} dt$

2. (15 pts) Let  $f(x) = x^4$  and  $b > 0$  be constant.

- (a) Find the average value of  $f(x)$  on the interval  $[0, b]$ .
- (b) Using your answer to part (a), find the appropriate value of  $c$  from the Mean Value Theorem for Integrals.

(c) Find the value of  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{2bi}{n}\right)^4 \frac{b}{n}$ .

3. (20 pts) The velocity of a particle moving along a line is  $v(t) = \sqrt{t} \cos(t)$ ,  $0 \leq t \leq 2\pi$ .

- (a) Approximate the displacement of the particle using  $M_2$ , the midpoint approximation with two equal subintervals.
- (b) Write (but do not evaluate) integral(s) to calculate the total distance traveled by the particle. Express your answer without using absolute value signs.

(c) Let  $g(x) = \int_0^x \sqrt{t} \cos t dt$ ,  $0 \leq x \leq 2\pi$ .

- i. On what intervals (if any) is  $g$  decreasing?
- ii. Find the value of  $g'(\pi)$ .

TURN OVER—More problems on the back!

4. (20 pts) The following problems are not related.

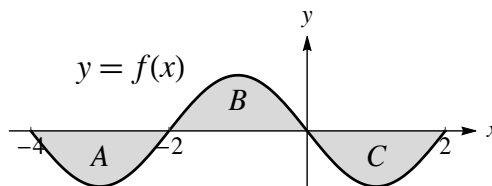
(a) Evaluate  $\sum_{k=10}^{100} k + \sum_{k=101}^{150} 2$ .

(b) Express the following in sigma notation and find the sum.

$$\ln(1/2) + \ln(2/3) + \ln(3/4) + \cdots + \ln(99/100)$$

(c) Each of the regions  $A$ ,  $B$ , and  $C$  bounded by the graph of  $f$  and the  $x$ -axis has an area of 7.

Find the value of  $\int_{-4}^2 (f(x) + 2x + 3) dx$ .



5. (24 pts) The following problems are not related.

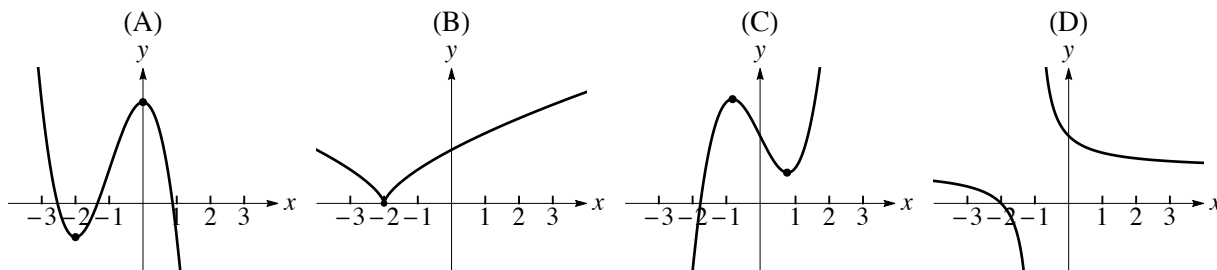
(a) The function  $h(x) = \frac{3}{2x+1}$  is one-to-one. Find  $h^{-1}$  and its range in interval notation.

(b) Suppose that  $g$  and  $G$  are functions for which  $G'(x) = g(x)$  and  $g$  is continuous. A table of values for  $G$  is shown below. Evaluate  $\int_4^9 \frac{g(\sqrt{x})}{\sqrt{x}} dx$  and simplify your answer fully.

$x$	0	1	2	3	4	5	9
$G(x)$	1	3	9	16	32	44	64

(c) Suppose Newton's Method is used to find the root(s) of a function  $f(x)$  given an initial approximation of  $x_1 = 0$ . The next two approximations have the values  $x_2 = 1$  and  $x_3 = 0$ .

- The graph of  $y = f(x)$  is shown below. Write the letter of the graph that matches  $f$ . No justification is necessary.
- Will the approximations  $x_4, x_5, \dots$ , converge toward a root of  $f$ ? Write Yes, No, or Maybe and explain your answer.



### Formulas

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$