

Print Name _____

APPM 1350

Exam 1

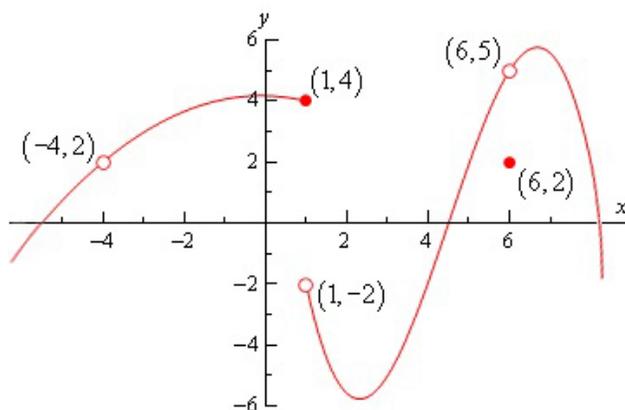
Fall 2016

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Include this exam sheet in your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Name any theorem that you use. Limit problems should not be evaluated using L'Hopital's Rule. Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

1. (16 points) Use the graph of f , shown below, to answer the following questions:

- What is $\lim_{x \rightarrow 1^+} f(x)$?
- What is $\lim_{x \rightarrow 6} f(x)$?
- State the definition of continuity of a function f at a number a .
- At what values of x does the function shown in the graph below fail to be continuous? Explain how each x -value fails to satisfy the definition of continuity. Be specific and provide details in your answer.



TURN OVER—More problems on the back!

2. (36 points) The following problems are not related.

(a) Find $\lim_{x \rightarrow 0} 2x \csc(5x)$.

(b) Find $\lim_{t \rightarrow -1} \frac{|t+1|}{t^2-1}$.

(c) Consider $\lim_{h \rightarrow 0} \frac{\sqrt{9+h}-3}{h}$.

i. Evaluate the limit.

ii. If the limit corresponds to the slope of the tangent line to a curve $y = f(x)$ at a point $(a, f(a))$, what is $f(x)$ and what is a ?

(d) Let $g(x) = x^3 - \cos(x) - 1$.

i. Show that g has a positive root. Justify your answer.

ii. Find the instantaneous rate of change of g at $x = 4\pi/3$.

3. (18 points) Let $h(x) = \frac{x^2 + 5x + 6}{2x^2 + 5x + 2}$.

(a) What is the domain of h ? Express your answer in interval notation.

(b) Find all horizontal asymptotes of h . Justify your answer using limits.

(c) Find all vertical asymptotes of h . Justify your answer using limits.

4. (18 points) Let $f(x) = 3x^2 - 5$.

(a) Use the limit definition of the derivative to calculate $f'(x)$.

(b) Find an equation for the line tangent to $y = f(x)$ at $x = -1$.

(c) Sketch a graph to illustrate the precise definition of $\lim_{x \rightarrow -1} (3x^2 - 5)$. Clearly label δ and ϵ in your graph.

5. (12 points) Sketch the graph of a single function g that satisfies all of the following conditions. No explanation is necessary.

g is odd

$$\lim_{x \rightarrow 2} g(x) = \infty$$

g is defined for all $x \neq \pm 2$

$$\lim_{x \rightarrow -\infty} g(x) = 0$$

$$\lim_{x \rightarrow 4} g(x) = -6$$

$$g(4) \neq \lim_{x \rightarrow 4} g(x)$$