INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your full name, (2) 1350/Final, (3) lecture number/instructor name and (4) SPRING 2018 on the front of your bluebook. Make a grading table for 5 problems and a total. Do all problems. Start each problem on a new page. Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Justify your answers, show all work.

1. (a)(13pts) (i) What is the domain of \( g(x) = \frac{x^2 + x}{x^2 + 3x + 2} \)? Give your answer in interval notation. (ii) Find all horizontal asymptotes of \( g(x) \), justify your answer with limits.

(b)(13pts) Find the real number \( a \) so that the function \( f(x) = \begin{cases} \ln(x + e^x + 2) & \text{if } x > 0 \\ a \cosh(x) & \text{if } x \leq 0 \end{cases} \) is continuous for all real numbers. Use limits to answer this question.

(c)(4pts) If \( y = (\tan^{-1}(x))^2 \) then \( dy/dx \) is equal to which of the options below? (No justification necessary - Choose only one answer, copy down the entire answer.)

\[
(A) \quad -2 \tan^{-3}(x) \sec^2(x) \\
(B) \quad \frac{2}{1 + x^2} \\
(C) \quad \frac{2 \tan^{-1}(x)}{1 + x^2} \\
(D) \quad 2 \arctan(x) \arcsin(x)
\]

2. (a)(15pts) What is the area of the largest rectangle in the first quadrant with two sides on the axes and one corner on the curve \( y = e^{-x} \)? Show all work and be sure to classify your answer either using the First Derivative Test or the Second Derivative Test.

(b)(15pts) Evaluate the definite integral \( \int_{1}^{\sqrt{3}} \frac{6}{1 + x^2} \, dx \). Simplify your answer.

(c)(5pts) If \( f(x) = \frac{\text{sech}^2(x)}{2 + \tanh(x)} \), then which of the choices below corresponds to \( \int f(x) \, dx \)? (No justification necessary - Choose only one answer, copy down the entire answer.)

\[
(A) \quad \frac{(2 + \tanh(x))^2}{2} + C \\
(B) \quad \ln(\tanh(x)) + C \\
(C) \quad \frac{2 \text{sech}(x) \tanh(x)}{2 + \tanh(x)} + C \\
(D) \quad \ln|2 + \tanh(x)| + C
\]

3. (a)(13pts) Use logarithmic differentiation to find the derivative of \( y = (\sec x)^{\ln x} \). Show all work.

(b)(13pts) Find the linearization of \( f(x) = \int_{0}^{\sin(x)} \sqrt{1 + t^2} \, dt \) centered at the point \( a = \pi \). Show all work.

(c)(4pts) The definite integral \( \int_{0}^{1} x e^{-2x^2} \, dx \) is equal to which choice below? (No justification necessary - Choose only one answer, copy down the entire answer.)

\[
(A) \quad \frac{1}{4}(1 - e) \\
(B) \quad \frac{1}{4}(e^{-2} - 1) \\
(C) \quad \frac{1 - e^{-2}}{4} \\
(D) \quad e^{-1} - e
\]
4. (a) (15 pts) Use l’Hospital’s Rule to evaluate the limit \( \lim_{x \to \infty} x \tan(1/x) \). Show all work.

(b) (15 pts) The half-life of Cesium-137 is 30 years. Suppose we initially have a 100-mg sample. (i) Find a formula for the mass remaining after \( t \) years. (ii) Set-up (but do not evaluate) an integral to calculate the average value of the mass remaining of Cesium-137 after 10 years.

(c) (5 pts) Which graph below most closely resembles the graph of \( g(x) = \frac{x^2 + x}{x^2 + 3x + 2} \)? (No justification necessary - Choose only one answer.)

5. (20 pts) Answer either ALWAYS TRUE or FALSE. You do NOT need to justify your answer. (Don’t just write down “A.T.” or “F”, completely write out the words “ALWAYS TRUE” or “FALSE” depending on your answer.)

(a) (5 pts) If the velocity of a particle at time \( t \) seconds is \( v(t) = 2t - 1 \) meters per second, then the total distance traveled during the time period \( 0 \leq t \leq 1 \) by the particle is 0.25 meters.

(b) (5 pts) By the Intermediate Value Theorem, the equation \( \log_2(x) + x = 0 \), for \( 0.5 \leq x \leq 4 \), has at least one root in the interval \((0.5, 4)\).

(c) (5 pts) If the function \( f(x) \) is continuous for all real values of \( x \) then \( f(x) \) is differentiable for all real values of \( x \).

(d) (5 pts) If \( f(x) = \ln(x) + \tan^{-1}(x) \) then \( f(1) = \pi/4 \) and \( (f^{-1})'(\pi/4) = 2/3 \).