APPM 1340 **Final Exam** **Fall 2017**

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor.
This exam is worth 150 points and has 7 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You will be taking this exam in a proctored and honor code enforced environment. This means: no notes or papers, calculators, cell phones, or other electronic devices are permitted.

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1. [30 pts] Evaluate the following limits. Be sure to show all your work.

   (a) $\lim\limits_{x\to\pi/8} \sin 2x \cos 2x \tan 2x$

   (b) $\lim\limits_{t\to-2^+} \dfrac{t^2-3t-10}{t^3+5t^2+6t}$

   (c) $\lim\limits_{x\to\infty} \dfrac{1-x^3}{3x+2}$

   (d) $\lim\limits_{\theta\to 0} \dfrac{\tan\theta}{\theta}$

   (e) $\lim\limits_{h\to 0} \dfrac{\sqrt{9+h}-3}{h}$

2. [40 pts] The following problems are unrelated.

   (a) Find the linearization of $y = \left(\dfrac{x+5}{x^2+2}\right)^2$ at $a = 1$.

   (b) Find $f''(\pi)$ if $f(x) = x\tan x$.

   (c) Let $y = 1 - |x-2|$.

      i. Write y as a piecewise function.

      ii. Graph y, labeling all intercepts.

      iii. Using your graph, find all x values, if any, where y is not differentiable.

   (d) Find the absolute extrema, if any exist, of the function $g(x) = \sqrt[3]{x^2}\,(2\sqrt[3]{x}-3)$ on $[-1, 1]$.

3. [10 pts] The height of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the height is 10 cm and the area is 100 cm^2?

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4. [15 pts] Consider the function $f(x) = x\sqrt{5-x}$.

   (a) Find the domain of $f(x)$, writing your answer using interval notation.

   (b) Find the average rate of change of $f(x)$ on the interval $[1, 5]$. What geometric property of the graph of f does the average rate of change represent?

   (c) Is there is a point in the interval $(1, 5)$ where the instantaneous rate of change of f equals its average rate of change over that interval? Justify your answer.

5. [25 pts] Suppose the position of a particle moving horizontally after t hours is given by $s(t) = \cos^2 t$ for $0 \leq t \leq 2\pi$. Let s be measured in miles with $s > 0$ corresponding to positions to the right of the origin.

   (a) Will the object ever be to the left of the origin? Briefly explain.

   (b) Find the velocity of the object at time t.

   (c) What time(s), if any, is the object at rest?

   (d) Find the total distance traveled by the object.

6. [15 pts] Consider the relation $x^2 + y^2 + 6x - 4y = -12$.

   (a) Draw the graph of this relation, labeling important points. (Hint: It is a circle)

   (b) Find $\mathrm{d}y/\mathrm{d}x$.

   (c) Find the point(s) on the graph where the slope of the tangent line is -1.

7. [15 pts] In your bluebook, write **T** if the statement is true and write **F** is the statement is false. No justification required and no partial credit given.

   (a) $\cos^2 \sqrt{x} + \sin^2 \sqrt{x} = 1$

   (b) Local extrema of a function occur at all of the function's critical points.

   (c) $q(a) = 0$ guarantees that the line $x = a$ is a vertical asymptote of the function $r(x) = \dfrac{p(x)}{q(x)}$.

   (d) If $\lim_{x\to a} f(x) \neq f(a)$, then $\lim_{x\to a} \dfrac{f(x) - f(a)}{x - a}$ does not exist.

   (e) If a function $f(x)$ is continuous on the interval $[a, b]$ and $f(a) = f(b)$, then f must possess a horizontal tangent at some point c in (a, b).