1. [12 pts] In your bluebook, write **T** if the statement is true and write **F** if the statement is false. No justification required and no partial credit given.

   (a) If x is a any real number whose square equals c, then x always equals the square root of c.

   (b) $\dfrac{\sec\theta}{\csc\theta} = \tan\theta$

   (c) $\cos^4 4x - \sin^4 4x = \cos 8x$

   (d) Let w, x, y, z represent positive quantities. Then $\sqrt{(xy)^2 + (w+z)^2} = xy + (w+z)$

   (e) $|x-4| \geq 4$ is equivalent to saying that x is in the set $(-\infty, 0] \cup [8, \infty)$

   (f) The lines $3x + 5y - 7 = 0$ and $6x + 10y + 4 = 0$ do not intersect.

   SOLUTION:

   (a) **FALSE** $x = \pm\sqrt{c}$ or $|x| = \sqrt{c}$.

   (b) **TRUE** $\dfrac{\sec\theta}{\csc\theta} = \dfrac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} = \dfrac{1}{\cos\theta}\sin\theta = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

   (c) **TRUE** $\cos^4 x - \sin^4 x = \left(\cos^2 4x + \sin^2 4x\right)\left(\cos^2 4x - \sin^2 4x\right) = 1\cos[2(4x)] = \cos 8x$

   (d) **FALSE** $\sqrt{a^2+b^2} \neq a + b$

   (e) **TRUE** $|x-4| \geq 4 \iff x - 4 \geq 4$ or $x - 4 \leq -4 \iff x \geq 8$ or $x \leq 0 \iff (-\infty, 0] \cup [8, \infty)$

   (f) **TRUE** The lines are parallel.

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2. [10 pts] Find the x- and y-intercepts of the line that passes through the point $(6, 5)$ and is perpendicular to the line that passes through the points $(-5, 2)$ and $(10, -7)$.

   SOLUTION:
   The line through the given points has slope $m = \frac{-7-2}{10-(-5)} = -\frac{3}{5}$. The slope of the line we are to find is the negative reciprocal of that line which is $\frac{5}{3}$. The line passing through $(6, 5)$ with slope $\frac{5}{3}$ has equation in point/slope form of $y - 5 = \frac{5}{3}(x-6)$. Rearranging this yields $3y - 15 = 5x - 30 \implies 5x - 3y = 15 \implies \frac{x}{3} - \frac{y}{5} = 1$. Thus the x-intercept is $(3, 0)$ and the y-intercept is $(0, -5)$. ■

3. [25 pts] Simplify the following expressions and eliminate any negative exponents. Assume that all letters denote positive numbers.

   a. $\left(\dfrac{2a^{-1}b}{a^2b^{-3}}\right)^{-3}$ b. $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}}$ c. $\left(2x^3y^{-1/4}\right)^2\left(8y^{-3/2}\right)^{-1/3}$ d. $\dfrac{2}{x} + \dfrac{3}{x-1} - \dfrac{4}{x^2-x}$ e. $\dfrac{1+\csc y}{\cos y + \cot y}$

   SOLUTION:

   (a) $\left(\dfrac{2a^{-1}b}{a^2b^{-3}}\right)^{-3} = \left(2a^{-3}b^4\right)^{-3} = 2^{-3}a^9b^{-12} = \dfrac{1}{2^3}a^9b^{-12} = \dfrac{a^9}{8b^{12}}$

   (b) $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}} = \dfrac{8^{1/3}x^{2/3}}{x^{1/2}} = 2x^{2/3-1/2} = 2x^{1/6}$

   (c) $\left(2x^3y^{-1/4}\right)^2\left(8y^{-3/2}\right)^{-1/3} = 2^2x^6y^{-2/4}8^{-1/3}y^{1/2} = 2x^6$

   (d) $\dfrac{2}{x} + \dfrac{3}{x-1} - \dfrac{4}{x^2-x} = \dfrac{2}{x} + \dfrac{3}{x-1} - \dfrac{4}{x(x-1)} = \dfrac{2(x-1)+3x-4}{x(x-1)} = \dfrac{5x-6}{x(x-1)}$

   (e) $\dfrac{1+\csc y}{\cos y + \cot y} = \left(\dfrac{1+\frac{1}{\sin y}}{\cos y + \frac{\cos y}{\sin y}}\right)\left(\dfrac{\sin y}{\sin y}\right) = \dfrac{\sin y + 1}{\cos y \sin y + \cos y} = \dfrac{\sin y + 1}{\cos y(\sin y + 1)} = \dfrac{1}{\cos y} = \sec y$

4. The following problems are not related.

(a) [4 pts] Rationalize the numerator of $\sqrt{x+1}-\sqrt{x}$.

(b) [5 pts] Factor $3x^{-1/2}+4x^{1/2}+x^{3/2}$ completely, writing your final answer without any fractional exponents.

(c) [5 pts] Solve the equation $3x^2-6x-1=0$ by completing the square.

(d) [5 pts] Evaluate $\sin(\theta-\phi)$ under the following conditions: $\tan\theta=\frac{4}{3}$, θ in Quadrant III, $\sin\phi=-\frac{\sqrt{10}}{10}$, ϕ in Quadrant IV. Recall that $\sin(u-v)=\sin u\cos v-\cos u\sin v$.

(e) [10 pts] Consider the function $f(x)=(x^4-10x^2+9)^{-1/2}$.

i. Find the domain of $f(x)$, writing your answer in interval notation.

ii. Is $f(x)$ even, odd or neither? Justify your answer.

iii. What symmetry, if any, does the graph of $f(x)$ possess?

SOLUTION:

(a) $\left(\sqrt{x+1}-\sqrt{x}\right)\left(\dfrac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\right)=\dfrac{x+1-x}{\sqrt{x+1}+\sqrt{x}}=\dfrac{1}{\sqrt{x+1}+\sqrt{x}}$

(b) $x^{-1/2}\left(3+4x+x^2\right)=x^{-1/2}\left(3+x\right)\left(1+x\right)=\dfrac{(x+3)(x+1)}{\sqrt{x}}$

(c) $3\left(x^2-2x\right)=1\Rightarrow 3\left(x^2-2x+1-1\right)=1\Rightarrow 3\left(x-1\right)^2=4\Rightarrow x-1=\pm\sqrt{\dfrac{4}{3}}\Rightarrow x=1\pm\dfrac{2\sqrt{3}}{3}$

(d) $\tan\theta=\frac{4}{3}$ with θ in Quadrant III implies that $y=-4$, $x=-3$ and from Pythagorean theorem $r=5$ so that $\sin\theta=\frac{y}{r}=-\frac{4}{5}$ and $\cos\theta=\frac{x}{r}=-\frac{3}{5}$. $\sin\phi=-\frac{\sqrt{10}}{10}$ with ϕ in Quadrant IV implies that $y=-\sqrt{10}$, $r=10$, and from Pythagorean theorem $x=3\sqrt{10}$. Thus $\cos\phi=\frac{3\sqrt{10}}{10}$. Therefore $\sin(\theta-\phi)=\sin\theta\cos\phi-\cos\theta\sin\phi=-\frac{4}{5}\left(\frac{3\sqrt{10}}{10}\right)-\left(-\frac{3}{5}\right)\left(-\frac{\sqrt{10}}{10}\right)=-\frac{15\sqrt{10}}{50}=\frac{-3\sqrt{10}}{10}$

(e) i. Domain consists of those points satisfying $x^4-10x^2+9>0$. But $x^4-10x^2+9=\left(x^2-1\right)\left(x^2-9\right)=(x+1)(x-1)(x+3)(x-3)=0$ if $x=-3,-1,1,3$. Partition the real line into the intervals $(-\infty,-3)$, $(-3,-1)$, $(-1,1)$ and $(3,\infty)$.

interval	$(-\infty,-3)$	$(-3,-1)$	$(-1,1)$	$(1,3)$	$(3,\infty)$
test value	-4	-2	0	2	4
$x+3$	$-$	$+$	$+$	$+$	$+$
$x+1$	$-$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$	$+$
$x-3$	$-$	$-$	$-$	$-$	$+$
x^4-10x^2+9	$+$	$-$	$+$	$-$	$+$

Note that $x^4-10x^2+9>0$ for x in $(-\infty,-3)$ or $(-1,1)$ or $(3,\infty)$ implying that the domain of $f(x)$ is $(-\infty,-3)\cup(-1,1)\cup(3,\infty)$.

ii. $f(-x)=[(-x)^4-10(-x)^2+9]^{-1/2}=(x^4-10x^2+9)^{-1/2}=f(x)$ so $f(x)$ is even.

iii. The graph of $f(x)$ is symmetric with respect to the y-axis.

5. [24 pts] Solve the following equations.

(a) $\sqrt{5-x}-x=-3$

(b) $\dfrac{x}{2x+7}-\dfrac{x+1}{x+3}=1$

(c) $3\tan^2\theta-1=0$, finding all solutions in the interval $[0,2\pi)$.

(d) $\cos 2\theta = -\sin\theta$, finding all solutions in the interval $[0, 2\pi)$.

SOLUTION:

(a) $\left(\sqrt{5-x}\right)^2 = (-3+x)^2 \Rightarrow 5-x = 9-6x+x^2 \Rightarrow 0 = 4-5x+x^2 \Rightarrow (x-4)(x-1) = 0 \Rightarrow x = 1, 4$. Checking in the original equation shows that $x = 1$ is an extraneous solution so the only solution to the equation is $x = 4$.

(b)

$$\left[\frac{x}{2x+7} - \frac{x+1}{x+3} = 1\right](2x+7)(x+3)$$

$$x(x+3) - (x+1)(2x+7) = (2x+7)(x+3)$$

$$x^2 + 3x - (2x^2 + 9x + 7) = 2x^2 + 13x + 21$$

$$3x^2 + 19x + 28 = 0$$

$$(3x+7)(x+4) = 0$$

$$3x + 7 = 0 \Rightarrow x = -\frac{7}{3} \text{ or } x + 4 = 0 \Rightarrow x = -4$$

(c) $3\tan^2\theta - 1 = 0 \Rightarrow \tan^2\theta = \frac{1}{3} \Rightarrow \tan\theta = \pm\frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(d) $\cos 2\theta = 1 - 2\sin^2\theta = -\sin\theta \Rightarrow 2\sin^2\theta - \sin\theta - 1 = 0 \Rightarrow (2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0 \Rightarrow \sin\theta = -\frac{1}{2} \Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}; \quad \sin\theta - 1 = 0 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

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