INSTRUCTIONS: Outside paper and electronic devices are **not** permitted. Exam is worth 100 points. Neatness counts. Unless indicated, answers with no supporting work may receive no credit. BOX your final answers.

1. (12 points) Choose the most appropriate answer for the following:

(a) $f(x) = \left[\dfrac{x(x-2)}{x^4 - 2x^3 + 100x^2 - 200x}\right] \implies f'(x) = ?$

(A) $\dfrac{-2x}{x^4 + 10,000}$ (B) $-(x^2+100)^2 2x$ (C)$-x^2 - 100$

(D) $\dfrac{-2x}{(x^2+100)^{-2}}$ (E)$\dfrac{1}{-(x^2+100)^2}$ (F) None of the Above

(b) $f(x) = \dfrac{1}{x^2 - 4x + 4} \implies f'(x) = ?$

(A) $\dfrac{-2}{(x-2)^3}$ (B) $\dfrac{x^2 - 6x + 8}{(x^2 - 4x + 4)^2}$ (C) $\dfrac{2x-4}{(x^2 - 4x + 4)}$

(D)$-(x^2 - 4x + 4)(2x - 4)$ (E) $-2(x-2)^3$ (F) None of the Above

(c) $f(t) = \left[\dfrac{\tan(6t)}{\sin(2t)}\right] \implies f'(t) = ?$

(A) $\dfrac{2\tan(6t)\cos(2t) - 6\sin(2t)\sec^2(6t)}{\sin^2(2t)}$ (B) $\dfrac{6\sin(2t)\sec^2(6t) - 2\tan(6t)\cos(2t)}{\sin^2(2t)}$

(C) $\dfrac{6\sin(2t)\sec(6t)\tan(6t) - 2\tan(6t)\cos(2t)}{\sin^2(2t)}$ (D) $\dfrac{2\tan(6t)\cos(2t) - 6\sin(2t)\sec(6t)\tan(6t)}{\sin^2(2t)}$

(E) $\dfrac{2\sin(6t)\sec^2(6t) - 6\tan(2t)\cos(6t)}{\sin^2(2t)}$ (F) None of the Above

Solution:
(a) F, $\dfrac{-2x}{(x^2+100)^2}$
(b) A
(c) B

2. The following problems are not necessarily related.

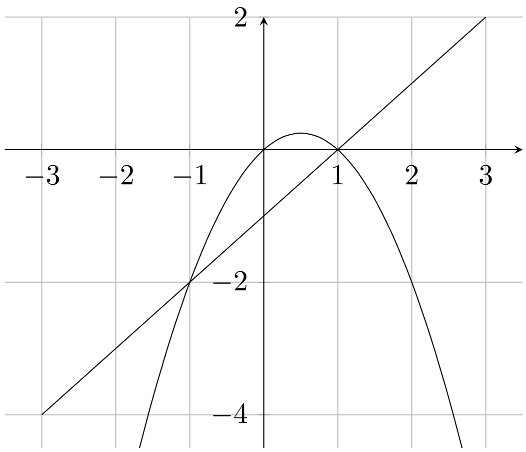
(a) (6 points) The position of a particle is $S(t) = -t^2 + 6t - 8$. Find the acceleration of the particle at 2.7 seconds.

(b) (12 points) Where does the normal line to the parabola $y = x - x^2$ at the point $(1, 0)$ intersect the parabola a second time? Illustrate with a sketch.

Solution:

(a) $a(t) = -2 \implies a(2.7) = \boxed{-2}$

(b) The second point of intersection is at $\boxed{(-1, -2)}$



3. (16 points) Consider the equation: $y = \sqrt{x}$.

(a) Find the instantaneous rate of change of y at $x = 16$.

(b) Find the point on the curve where the tangent has slope $\frac{1}{6}$.

Solution:

(a) $y' = \dfrac{1}{2\sqrt{x}} \implies y'(16) = \boxed{\dfrac{1}{8}}$

(b) $y' = \dfrac{1}{2\sqrt{x}} = \dfrac{1}{6} \implies 2\sqrt{x} = 6 \implies \sqrt{x} = x \implies x = 9 \implies$ point $\boxed{(9, 3)}$

4. (16 points) Find the following derivatives:

(a) $f(\theta) = \left[\dfrac{\theta\cos^2(\theta) - \theta}{\theta}\right] \implies f'(\theta) = ?$ (b) $f(x) = \left[\dfrac{x^2 + x - 6}{|x-2|}\right] \implies f'(x) = ?$

Solution:

(a) $f'(\theta) = (-\sin^2\theta)' = -2\sin\theta\cos\theta = \boxed{-\sin(2\theta)}$

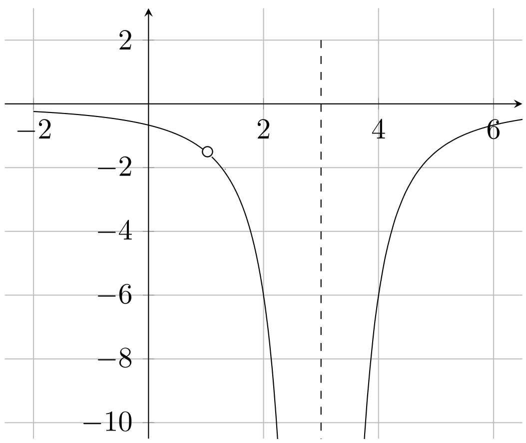
(b) $f(x) = \begin{cases} (x+3) & , x > 2 \\ -(x+3) & , x < 2 \end{cases}$ so, $\boxed{f'(x) = \begin{cases} 1 & , x > 2 \\ -1 & , x < 2 \end{cases}}$

5. (6 points)
Consider a function, $f(x)$, with all six of the following characteristics. Sketch $f'(x)$.

(i) $f(1) = \emptyset$ (ii) $\lim_{x \to 1} [f(x)] = -1$ (iii) $\lim_{x \to 3^+} [f(x)] = \infty$

(iv) $\lim_{x \to 3^-} [f(x)] = -\infty$ (v) $\lim_{x \to -\infty} [f(x)] = 2$ (vi) $\lim_{x \to \infty} [f(x)] = 2$

Solution:



6. Consider the following problem and answer the questions below:
If a snowball melts so that its surface area decreases at a rate of 1 $\frac{cm^2}{min}$, find the rate at which the diameter decreases when the diameter is 10 cm.

(a) (2 points) Draw a picture, with labels, of this situation for any time t.

(b) (2 points) Using mathematical notation, what are the given values in this question?

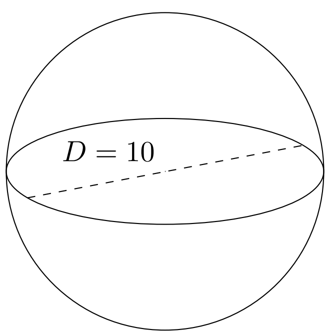
(c) (2 points) Using mathematical notation, what are you looking for?

(d) (2 points) Write an equation that relates surface area and diameter.

(e) (2 points) Write an equation that relates the rate of change of surface area to the rate of change of diameter.

(f) (6 points) What is the rate at which the diameter decreases when the diameter is at 10 cm.?.

Solution:
(a) S = Surface Area and D = Diameter



(b) $\boxed{\frac{dS}{dt} = -1 \text{ and } D = 10}$

(c) $\boxed{\left.\frac{dD}{dt}\right|_{D=10}}$

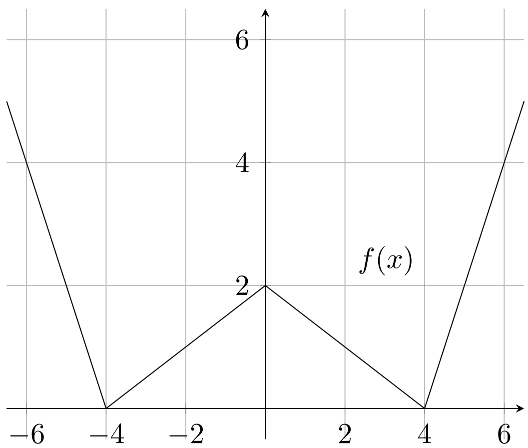
(d) $S = 4\pi r^2 \implies S = 4\pi\left(\frac{D}{2}\right)^2 \implies \boxed{S = \pi D^2}$

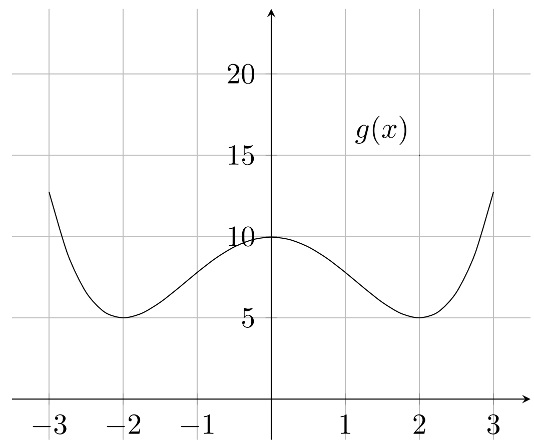
(e) $\boxed{\frac{dS}{dt} = 2\pi D\frac{dD}{dt}}$, or $\boxed{\frac{dD}{dt} = \frac{1}{2\pi D}\cdot\frac{dS}{dt}}$

(f) $\left.\frac{dD}{dt}\right|_{D=10} = \frac{1}{2\pi(10)}\cdot(-1) = \boxed{\frac{-1}{20\pi}}$

7. (16 points) Consider the graphs of $f(x)$ and $g(x)$ below (No work required).

(a) Find $f'(-3) + f'(1)$ (b) Find $[f^2(-2)]'$ (c) Find $f'(g(2))$ (d) Find $(fg)'(2)$





Solution:

(a) 0

(b) 0, as opposed to $(f^2)'(-2) = 1$

(c) 2

(d) $-\frac{5}{2}$