INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. This exam is worth 100 points. Box your final answers. Write neatly, top to bottom, left to right, one problem per page. A correct answer with incorrect or no supporting work may receive no credit. If you need to find a derivative then you must find it via definition. **SHOW ALL WORK**

1. (20 points) Evaluate the following limits:

   (a) $\lim_{\theta\to 0}\left[\dfrac{\theta\cos^2(\theta)+10\theta^2-\theta}{\theta^2}\right]$ (b) $\lim_{x\to 2}\left[\dfrac{\sqrt{4x+1}-3}{x-2}\right]$ (c) $\lim_{t\to 0}\left[\dfrac{\tan(6t)}{\sin(2t)}\right]$ (d) $\lim_{x\to 2^-}\left[\dfrac{x^2+x-6}{|x-2|}\right]$

2. (8 points) Consider the function: $f(x)=\begin{cases} -x^2+6x-8 & ,x>3 \\ x-2 & ,x<3 \\ 1 & ,x=3 \end{cases}$

   Show that $f(x)$ is either continuous on the real numbers, or name any points of discontinuity.

3. (15 points) Consider the equation: $y=\sqrt{x}$.

   (a) Find the average rate of change of y between $x=1$ and $x=4$.

   (b) Find the instantaneous rate of change of y at $x=1$.

   (c) Find the equation of the tangent line to the curve y at $x=1$.

4. (12 points) The following may not be related:

   (a) $f(x)=\dfrac{2}{x}$, then $f'(1)=?$ (b) $\lim_{x\to\infty}\left[\dfrac{x(2x^2-8)}{x^3-2x^2+100x-200}\right]=?$

5. (8 points) Find $\lim_{x\to 0}\left[\dfrac{1-\cos(x)}{x^2}\right]$, given $-\dfrac{x^2}{24}<\dfrac{2-2\cos(x)-x^2}{2x^2}$ and $\dfrac{x^2}{2}>[1-\cos(x)]$.
   (Full credit is awarded for using the given information)

6. (12 points)
   (a) Sketch a function with all six of the characteristics listed below.
   (b) Create a function with all six of the characteristics listed below.

   (i) $f(1)=\emptyset$ (ii) $\lim_{x\to 1}[f(x)]=-1$ (iii) $\lim_{x\to 3^+}[f(x)]=\infty$

   (iv) $\lim_{x\to 3^-}[f(x)]=-\infty$ (v) $\lim_{x\to-\infty}[f(x)]=2$ (vi) $\lim_{x\to\infty}[f(x)]=2$

MORE ON THE BACK

7. (15 points) Explain why the following statements are true or false.
Consider a number of ideas in your explanation: graphs, continuity, increasing functions, the IVT etc.
Grading on this problem is dependent on neatness, thoroughness and succinctness of explanation.

(a) The following statement is true, explain why:
$f(x) = x^2 - 4x + 7$ equals π somewhere between $x = 0$ and $x = 6$.

(b) The following statement is false, explain why:
The I.V.T. can be used to show $f(x) = \dfrac{x^3 + 8x + 10}{x - 1}$ equals 10 somewhere between $x = 0$ and $x = 2$.

(c) The following statement is true, explain why:
$f(x) = \dfrac{8x + 10}{4x^2 + x - 5}$ equals 3 somewhere between $x = 0$ and $x = 2$.

8. (10 points) Given the following graphs of $f(x)$ and $g(x)$, sketch the graphs of $f'(x)$ and $g'(x)$.
No explanation required. You can use the axis system provided as scratch paper, but you must reproduce a sketch of your graphs in your blue book for any credit.