INSTRUCTIONS: Books, notes, and electronic devices are not permitted. This exam is worth 100 points. Box your final answers. Write neatly, top to bottom, left to right, one problem per page. A correct answer with incorrect or no supporting work may receive no credit. If you need to find a derivative then you must find it via definition. SHOW ALL WORK

1. (20 points) Evaluate the following limits:
   
   (a) \( \lim_{\theta \to 0} \left[ \frac{\theta \cos^2(\theta) + 10\theta^2 - \theta}{\theta^2} \right] \)
   
   (b) \( \lim_{x \to 2} \left[ \frac{\sqrt{4x + 1} - 3}{x - 2} \right] \)
   
   (c) \( \lim_{t \to 0} \left[ \frac{\tan(6t)}{\sin(2t)} \right] \)
   
   (d) \( \lim_{x \to 2^-} \left[ \frac{x^2 + x - 6}{|x - 2|} \right] \)

2. (8 points) Consider the function:
   
   \[ f(x) = \begin{cases} 
   -x^2 + 6x - 8 & , x > 3 \\
   x - 2 & , x < 3 \\
   1 & , x = 3 
   \end{cases} \]

   Show that \( f(x) \) is either continuous on the real numbers, or name any points of discontinuity.

3. (15 points) Consider the equation: \( y = \sqrt{x} \).

   (a) Find the average rate of change of \( y \) between \( x = 1 \) and \( x = 4 \).

   (b) Find the instantaneous rate of change of \( y \) at \( x = 1 \).

   (c) Find the equation of the tangent line to the curve \( y \) at \( x = 1 \).

4. (12 points) The following may not be related:

   (a) \( f(x) = \frac{2}{x} \), then \( f'(1) = ? \)

   (b) \( \lim_{x \to \infty} \left[ \frac{x(2x^2 - 8)}{x^3 - 2x^2 + 100x - 200} \right] = ? \)

5. (8 points) Find \( \lim_{x \to 0} \left[ \frac{1 - \cos(x)}{x^2} \right] \), given \( \frac{-x^2}{24} < \frac{2 - 2\cos(x) - x^2}{2x^2} \) and \( \frac{x^2}{2} > |1 - \cos(x)| \).

   (Full credit is awarded for using the given information)

6. (12 points)

   (a) Sketch a function with all six of the characteristics listed below.

   (b) Create a function with all six of the characteristics listed below.

   \( (i) \ f(1) = \emptyset \quad (ii) \lim_{x \to 1} [f(x)] = -1 \quad (iii) \lim_{x \to 3^+} [f(x)] = \infty \)

   \( (iv) \lim_{x \to 3^-} [f(x)] = -\infty \quad (v) \lim_{x \to -\infty} [f(x)] = 2 \quad (vi) \lim_{x \to \infty} [f(x)] = 2 \)

MORE ON THE BACK
7. (15 points) Explain why the following statements are true or false. Consider a number of ideas in your explanation: graphs, continuity, increasing functions, the IVT etc. Grading on this problem is dependent on neatness, thoroughness and succinctness of explanation.

(a) The following statement is true, explain why:
\[ f(x) = x^2 - 4x + 7 \] equals \( \pi \) somewhere between \( x = 0 \) and \( x = 6 \).

(b) The following statement is false, explain why:
The I.V.T. can be used to show \( f(x) = \frac{x^3 + 8x + 10}{x - 1} \) equals 10 somewhere between \( x = 0 \) and \( x = 2 \).

(c) The following statement is true, explain why:
\[ f(x) = \frac{8x + 10}{4x^2 + x - 5} \] equals 3 somewhere between \( x = 0 \) and \( x = 2 \).

8. (10 points) Given the following graphs of \( f(x) \) and \( g(x) \), sketch the graphs of \( f'(x) \) and \( g'(x) \).
No explanation required. You can use the axis system provided as scratch paper, but you must reproduce a sketch of your graphs in your blue book for any credit.