1. (20 points) Short Answer. Show all work.

   a. Determine the quadrant in which each angle lies:

      (i) \(-\frac{3\pi}{8}\)   \[\text{Quadrant V}\]

      (ii) \(\frac{11\pi}{9}\)   \[\text{Quadrant III}\]

      (iii) \(\frac{21\pi}{4}\)  \[\text{Quadrant III}\]

   b. Evaluate the expression \((\sin(30°))^2 + (\cos(30°))^2\)

   c. Evaluate the following:

      (i) \(\sin(270°)\) \[\frac{-1}{2}\]

      (ii) \(\cos\left(\frac{3\pi}{4}\right)\) \[\frac{-\sqrt{2}}{2}\]

      (iii) \(\tan\left(-\frac{2\pi}{3}\right)\) \[\sqrt{3}\]

   d. Find the reference angle for \(\theta = -105°\) \[75°\]

   e. Find the terminal point on the unit circle in \((x, y)\) coordinates for \(\tau = \frac{11\pi}{6}\) \(\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)\)
2. (14 points) Find the exact values of the six trigonometric functions of $\theta$ if $\theta$ is in standard position and the terminal side of $\theta$ lies in Quadrant III on the line $3y - 4x = 0$.

$$y = \frac{4}{3} x$$

$$\sin \theta = -\frac{4}{5}$$
$$\cos \theta = -\frac{3}{5}$$
$$\tan \theta = \frac{4}{3}$$
$$\csc \theta = -\frac{5}{4}$$
$$\sec \theta = -\frac{5}{3}$$
$$\cot \theta = \frac{3}{4}$$

3. (16 points) The temperature of a car engine after it is turned off is modeled by the equation $\log \left( \frac{F-20}{200} \right) = -0.4t$, where $F$ is the temperature in degrees Fahrenheit and $t$ is the time in minutes after the engine is turned off. Give your answer in the correct units: Work must be shown. Trial and error will not be accepted for sufficient work.

a. Solve the given equation for $F$ to obtain an equation for $F$ in terms of time $t$.

Exponential Form:

$$10^{-0.4t} = \frac{F-20}{200}$$

$$200(10^{-0.4t}) = F - 20$$

$$\Rightarrow F = 20 + 200 \cdot 10^{-0.4t}$$

b. At what time $t$ does the temperature of the engine reach 40°F?

Set $F = 40$

$$\log \left( \frac{40-20}{200} \right) = \log \left( \frac{1}{10} \right) = -1$$

So we have

$$-1 = -0.4t$$

$$\Rightarrow t = 2.5 \text{ min}$$

c. What is the temperature of the engine when it is turned off?

$t = 0$

From part (a)

$$F = 20 + 200 \cdot 10^0 = 20 + 200 \cdot 1$$

$$= 220^\circ F$$

d. What temperature does the engine approach as $t \to \infty$. Your answer should be a number.

$$F = 20 + 200 \cdot 10^{-0.4t}$$

\[\text{at } t \to \infty, \quad 10^{-0.4t} \to 0\]

So

$$F = 20 + 200 \cdot 0 = 20^\circ F$$
4. (10 points) Write as a single logarithm. 

\[
\log \left( \frac{a^6 b^8}{a^2 b^2 a} \right) = \log \left( a^3 b^4 \right)
\]

\[
3 \log(a^2 b^3) - 2 \log(ab) + \log \left( \frac{1}{a} \right)
\]

5. (16 points) Solve the following for \( x \):

a. \( 3 \ln \left( \frac{1}{x} \right) - x = 0 \)

\[x \left[ 3 \ln \left( \frac{1}{x} \right) - 1 \right] = 0\]

\[x = 0 \quad \text{or} \quad \ln \left( \frac{1}{x} \right) = \frac{1}{3}\]

\[x = e^{-\frac{1}{3}}\]

b. \( e^x + 2 = 8 e^{-x} \)

\[e^x + 2 - 8e^{-x} = 0\]

\[e^{-x}(e^x + 2e^x - 8) = 0\]

\[e^{-x}(e^x + 4)(e^x - 2) = 0\]

\( e^{-x} = 0 \quad \emptyset \)

\( e^x + 4 = 0 \quad \Rightarrow e^x = -4 \quad \emptyset \)

\( e^x - 2 = 0 \quad \Rightarrow e^x = 2 \quad \Rightarrow x = \ln 2\]

6. (6 points) A circular arc of length 10 feet subtends a central angle of 45°. Find the radius of the circle.

\[S = r \theta\]

\[10 \text{ ft} = r \cdot \frac{\pi}{4}\]

\[r = \frac{40}{\pi} \text{ ft}\]
7. (6 points) A cyclist is riding a bicycle whose wheels have a diameter of 24 inches. Suppose the wheels turn at a rate of 30 revolutions per minute. Find the speed of the cyclist in feet per minute.

\[ v = 1 \text{ ft} \]

\[ 30 \text{ rev/min} \cdot \frac{2\pi}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{1 \text{ ft}} = 60\pi \text{ ft/min} \]

8. (6 points) Choose the equation below that describes the graph: A cosine curve with a period of 4 \( \pi \), an amplitude of 2, a right phase shift of \( \frac{\pi}{2} \) and a vertical translation up 3 units

(a) \( f(x) = 2 \cos \left( \frac{\pi}{2} x - \frac{1}{4} \right) + 3 \)
(b) \( f(x) = 3 \cos \left( \frac{\pi}{2} x - \frac{\pi}{2} \right) + 2 \)
(c) \( f(x) = 2 \cos \left( \frac{\pi}{2} x - \frac{\pi}{4} \right) + 3 \)
(d) \( f(x) = 2 \cos \left( \frac{\pi}{2} x + \frac{\pi}{4} \right) + 3 \)
(e) \( f(x) = 2 \cos \left( \frac{\pi}{2} x - \frac{1}{4} \right) - 3 \)

9. (6 points) Match the trigonometric function with one of the graphs I-VI: \( f(x) = \tan \left( x + \frac{\pi}{4} \right) \)

CIRCLE THE CORRECT GRAPH.