APPM 1235 Final Exam  Spring 2016
May 2, 2016

AT THE TOP OF THE PAGE write your name and your section number. The following items are NOT permitted to be used during this exam: textbooks, class notes, other notes or study materials of any kind, slide rules, and electronic devices of any kind including but not limited to calculators and mobile phones. Any scratch paper used during this exam must be only that which has been provided by the proctor(s).

Do all your work on this exam. Any work done on scrap paper will NOT be graded.

Part I.  Show Your Work, 8 problems, 90 points
Part II.  Short Answer, 12 problems, 60 points  Total: 150 points

Part I. Show Your Work. Simplify all solutions. Method counts-- for most of these problems you must show a complete and valid solution method for full credit. Leave your answers in terms of π as necessary. Use interval notation as necessary. BOX your final answers when an answer line is not provided.

1. [12 points] The voltage in millivolts (mV) of an RC circuit as a function of time in milliseconds (ms) is given by the function $V(t) = 20 \left(1 - e^{-2t}\right)$ for $t \geq 0$.

(a) What is $V(0)$? ____________________________

(b) What voltage $V$ does the circuit approach as $t \to \infty$? ____________________________

(c) At what time $t$ will $V = 5$ mV?

$$5 = 20 \left(1 - e^{-2t}\right)$$

$$\frac{1}{4} = 1 - e^{-2t}$$

$$e^{-2t} = \frac{3}{4}$$

$$-2t = \ln \left(\frac{3}{4}\right)$$

$$t = -\frac{1}{2} \ln \left(\frac{3}{4}\right)$$
2. [12 points] Answer the following concerning the function \( P(x) = \frac{1}{4}(x - 2)^2(x + 5)(x - 4) \):

(a) Where is \( P(x) > 0 \)?

\[ (-\infty, -5), (4, \infty) \]

(b) A function \( Q(x) \) is defined to be \( Q(x) = 5P(x - 3) \). What are the zeros of \( Q(x) \)?

\[ x = -2, 5, 7 \]

3. [12 points] Answer the following concerning the function \( g(x) = x + 4 + \frac{8}{x - 5} \).

(a) At what values of \( x \) does \( g(x) = 0 \), if any?

\[
x + 4 + \frac{8}{x - 5} = 0
\]

\[
x^2 - x - 12
\]

\[
x - 5
\]

\[
(x - 4)(x + 3)
\]

\[
x - 5
\]

\[ x = 4, -3 \]

(b) State all the asymptotes (horizontal, vertical and/or slant) of \( g(x) \).

\[
SA : y = x + 4
\]

\[
VA : x = 5
\]
4. [10 points] Find all solutions: \( \log_8 x^3 + \log_2 (x - 6) = \log_2 16 \)

\[
\frac{\log_2 x^3}{\log_2 8} + \log_2 (x - 6) = \log_2 16 \\
\frac{1}{3} \log_2 x^3 + \log_2 (x - 6) = \log_2 16 \\
\log_2 x + \log_2 (x - 6) = \log_2 16 \\
\log_2 (x^2 - 6x) = \log_2 16 \quad \rightarrow \quad x^2 - 6x = 16 \\
x^2 - 6x - 16 = 0 \\
(x - 8)(x + 2) = 0 \\
x = 8, > \underline{8} \\
\underline{x = 8}
\]

5. [10 points] Factor completely and simplify: \( \frac{3}{2} x^2 (x^2 + 8)^{-1/2} - \frac{1}{2} (x^2 + 8)^{1/2} \). Leave no negative exponents in the resulting expression.

\[
\frac{1}{2} \left( x^2 + 8 \right)^{-1/2} \left[ 3x^2 - (x^2 + 8) \right] \\
= \frac{2x^2 - 8}{2(x^2 + 8)^{1/2}} \\
= \frac{(x+2)(x-2)}{(x^2 + 8)^{1/2}}
\]
6. [10 points] Set up and then simplify the difference quotient \( \frac{f(x+h) - f(x)}{h} \) for \( f(x) = \frac{x}{x+2} \).

\[
\begin{align*}
DQ &= \frac{1}{h} \left[ \frac{x+h}{x+h+2} - \frac{x}{x+2} \right] \\
&= \frac{1}{h} \left[ \frac{x^2 + xh + 2x + 2h - x^2 - xh - 2x}{(x+h+2)(x+2)} \right] \\
&= \frac{-2}{(x+h+2)(x+2)}
\end{align*}
\]

7. [12 points] A ball is thrown straight up from the top of 128-foot cliff and falls to the ground below the cliff. The ball’s height above the ground in feet as a function of time in seconds is determined by the function \( h(t) = -16t^2 + 32t + 128 \). Answer the following questions giving the correct units with your answers.

(a) Determine the average rate of change of the height of the ball from the time that it is thrown to the time it hits the ground.

\[ h(t) = -16(t^2 - 2t - 8) = -16(t-4)(t+2) \]

\[ h(0) = 128 \]

\[ h(4) = 0 \]

Average Rate of Change (ARC) = \( \frac{0-128}{4-0} = \boxed{-32 \text{ ft/s}} \)

(b) What is the maximum height the ball reaches?

\[ \max h \text{ at } t = 4 < \left( \frac{-b}{2a} = \frac{-32}{-32} = 1 \right) \]

\[ h(1) = -16 + 32 + 128 = 144 \text{ ft} \]
8. [12 points] A boat is cruising the ocean off a straight shoreline. Points A and B are 120 miles apart on the shore as shown. It is found that angle A is 45° and angle B is 60°. Find the shortest distance \( x \) from the boat to the shore.

\[
\tan 60^\circ = \frac{x}{120-x} = \sqrt{3}
\]

\[
x = 120\sqrt{3} - \sqrt{3}x
\]

\[
x + \sqrt{3}x = 120\sqrt{3}
\]

\[
x = \frac{120\sqrt{3}}{1 + \sqrt{3}}
\]

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Part II. Short Answer  [5 points each, 60 points total] These problems will be graded on your answers only. Give your answers on the blank lines on the right side of the page.

9. Solve: \( \ln x = e \)

\[ \boxed{e} \]

10. Find all solutions on \([0, 2\pi)\): \( \cos 3\theta = 3 \)

\[ \text{No sol'n} \]

11. Find all solutions: \( (2x-1)^2 - 5(2x-1) + 4 = 0 \)

\[
\begin{align*}
(2x-1)^2 - 5(2x-1) + 4 &= 0 \\
(2x-1)(2x-1) - 5(2x-1) + 4 &= 0 \\
(2x-1)(2x-1) - 5(2x-1) + 4 &= 0 \\
2x-1 &= 1 \Rightarrow x = 1 \\
2x-1 &= 4 \Rightarrow x = \frac{5}{2} \\
2x-1 &= 5 \Rightarrow x = 3
\end{align*}
\]

\[ 1, \frac{5}{2} \]

12. Find the exact value of \( \cos^{-1}\left(\sin \frac{4\pi}{3}\right) \)

\[ = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6} \]

13. Simplify: \( \frac{b-a}{1} \). Choose the correct answer:

\[ \frac{b^2-a^2}{b-a} = b+a \]

A. \( a^2-b^2 \)  B. \( b-a \)  C. \( \frac{b^2-a^2}{ab} \)  D. \( a+b \)  E. \( \frac{(b-a)^2(b+a)}{a^2b^2} \)

\[ \boxed{D} \]
14. The function \( f(x) = x^3 + 6x^2 + 3x - 10 \) has a factor of \((x - 1)\). Find all the real zeros of the function.
\[
\begin{array}{c|ccc}
 & 1 & 6 & 3 - 10 \\
\hline 
1 & -5 & -4 \\
\hline 
 & -7 & 10 \\
\end{array}
\]

\[ x^2 + 7x + 10 = (x + 5)(x + 2) \]

14. \(-5, -2, 1\)

15. Consider the following five functions: \( y = \tan^{-1} x, \ y = e^x, \ y = |x|, \ y = \sqrt[3]{x}, \) and \( y = \frac{1}{x^2 + 1} \). Which one of the following is true for all five functions?

A. has a horizontal asymptote  
B. passes through \((0, 0)\)  
C. domain is all real numbers  
D. is increasing on \(x > 0\)  
E. has neither odd nor even symmetry

15. \( C \)

16. State the domain of the function \( y = \frac{\sin x}{x} \).

16. \( x \neq 0 \)

17. True or false: \( \left(16a^2 - 2b^2\right)^{1/2} = 4a - \sqrt{2}b \). Circle the correct answer.

17. True \[ \text{False} \]

18. For \( f(x) = x^2 - 6x + 9 \) for \( x \geq 3 \), find \( f^{-1}(9) \).
\[
x^2 - 6x + 9 = 9 \\
x^2 - 6x = 0 \\
x = 0, 6
\]

18. \( 6 \)

19. Given \( g(x) = x^2 \) and \( h(x) = \frac{1}{\sqrt{2x - 1}} \), which one of the following is the domain of \( g \circ h \)?

A. \( x > \frac{1}{2} \)  
B. \( x \geq \frac{1}{2} \)  
C. \( x \neq \frac{1}{2} \)  
D. \((-\infty, \infty)\)  
E. \( x > \frac{1}{\sqrt{2}} \) or \( x < -\frac{1}{\sqrt{2}} \)

19. \( A \)

20. Which one of the following most accurately describes the function \( y = \sin x \) ?

A. \( -\frac{\pi}{2} \leq \sin x \leq \frac{\pi}{2} \)  
B. \( 0 \leq \sin x \leq 2\pi \)  
C. \( -1 \leq \sin x \leq 1 \)  
D. \( 0 \leq \sin x \leq 1 \)  
E. \( -\infty \leq \sin x \leq \infty \)

20. \( C \)