1. [8, 8, 8 points] Answer the following concerning the function \( g(x) = \frac{5}{6-x} \).

(a) Find the difference quotient \( DQ \) for \( g(x) \) where \( DQ = \frac{g(a+h) - g(a)}{h} \).

\[
DQ = \frac{1}{h} \left[ \frac{5}{6-a-h} - \frac{5}{6-a} \right] \\
= \frac{1}{h} \left[ \frac{30 - 5a - 30 + 5a + 5h}{(6-a-h)(6-a)} \right] \\
= \frac{5}{(6-a-h)(6-a)}
\]

(b) Find and simplify \( (g \circ g)(x) \).

\[
g \circ g = \frac{5}{6-x} \cdot \frac{6-x}{6-x} \\
= \frac{30 - 5x}{36 - 6x - 5} = \frac{30 - 5x}{31 - 6x}
\]

(c) Where is \( g(x) > -\frac{1}{3} \)? Give your answer in interval notation.

\[
\frac{5}{6-x} > -\frac{1}{3} \\
\frac{5}{6-x} + \frac{1}{3} > 0 \\
\frac{15 + 6-x}{3(6-x)} > 0
\]

\[\therefore (-\infty, 6) \cup (21, \infty)\]
2. [8, 2 points] Answer the following concerning the function $f(x) = x^2 - 4x$.

(a) Find and simplify an expression for the average rate of change of $f(x)$ from $x = 3$ to $x = k$.

$$\text{ARC} = \frac{f(k) - f(3)}{k - 3}$$

$$= \frac{k^2 - 4k - (3)}{k - 3}$$

$$= \frac{k^2 - 4k + 3}{k - 3}$$

$$= \frac{(k - 3)(k - 1)}{k - 3}$$

$$= k - 1$$

(b) Use your expression from part (a) to find $k$ such that the average rate of change is 5.

$$k - 1 = 5$$

$$\therefore k = 6$$
3. [8, 8, 2 points] These questions are not related.

(a) Factor completely and simplify: \( \frac{(3x^2 - 2)^{1/2}(3x^2) - x^{3/2}(3x^2 - 2)^{-1/2}(6x)}{3x^2 - 2} \). Leave no negative exponents in the resulting expression.

\[
\begin{align*}
&= \frac{(3x^2 - 2)^{1/2} \cdot 3x^2[(3x^2 - 2) - x^2]}{(3x^2 - 2)} \\
&= \frac{3x^2 \cdot (2x^2 - 2)}{(3x^2 - 2)^{3/2}} \\
&= \frac{3x^2 \cdot 2(x^2 - 1)}{(3x^2 - 2)^{3/2}} \\
&= \frac{6x^2(x + 1)(x - 1)}{(3x^2 - 2)^{3/2}}
\end{align*}
\]

(b) Solve: \( \frac{x}{x-2} + \frac{1}{x+2} = \frac{8}{x^2 - 4} \)

\[
x(x+2) + (x-2) = 8 \\
x^2 + 3x - 10 = 0 \\
(x+5)(x-2) = 0 \\
x = -5, 2
\]

(c) Which of the following is the function \( y = |x - 4| - 3 \) written as a piecewise function?

A. \( y = \begin{cases} x - 7 & x \geq 7 \\ -x + 7 & x < 7 \end{cases} \)

B. \( y = \begin{cases} x + 1 & x \geq 0 \\ -x - 1 & x < 0 \end{cases} \)

C. \( y = \begin{cases} x - 7 & x \geq 4 \\ -x + 1 & x < 4 \end{cases} \)

D. \( y = \begin{cases} x - 7 & x \geq 0 \\ -x + 1 & x < 0 \end{cases} \)

\[
y = \begin{cases} x - 7 & x \geq 4 \\ -x + 4 - 3 & x < 4 \end{cases}
\]}
4. [6, 6, 6 points] A new toll road between Boulder and Denver has opened and drivers are charged $3 for a trip between the two cities. To save money, commuters have two options for purchasing tolls at a discount:

- Deal A charges drivers $1.50 per trip if they purchase a 3-month pass for $18.
- Deal B allows unlimited trips between the two cities if drivers purchase a $48 monthly pass.

Let $x$ be the number of trips a driver will make per month.

(a) Using Deal A, express the monthly cost of traveling between the two cities as a function of $x$. Call this function $A(x)$.

$$A(x) = 6 + 1.5x$$

(b) Using Deal B, express the monthly cost of traveling between the two cities as a function of $x$. Call this function $B(x)$.

$$B(x) = 48$$

(c) For how many trips is Deal B a better deal than Deal A?

$$6 + 1.5x = 48$$

$$1.5x = 42$$

$$x = 28$$

So, 29 or more trips
5. [30 points total] This problem will be graded on your answers only.

The function shown below is $h(x)$. Point A on function $h(x)$ has the coordinates $(\sqrt{3},12)$.

(a) State the domain and range of $h(x)$ in interval notation.

$$D: [-3, 3]$$

$$R: [-12, 12]$$

(b) Which of the following statements is/are true about $h(x)$?

- A. $h(-x) = h(x)$
- B. $h(-x) = -h(x)$
- C. $h(x) = -h(x)$
- D. None of the above are true.

A new function is defined as $j(x) = \frac{-1}{4}h(2x)$.

(c) State the domain of $j(x)$ in interval notation.

$$\left[\frac{-3}{2}, \frac{3}{2}\right]$$

(d) What are the new coordinates of point A when the function $h(x)$ is transformed to $j(x)$?

$$\left(\frac{\sqrt{3}}{2}, -3\right)$$

(e) Another new function is defined as $k(x) = |h(x)|$. Sketch the function and label the $(x, y)$ coordinates of three points on $k(x)$. 

$$(-3, 0), (0, 0), (3, 0)$$