• Textbooks, class notes and electronic devices of any kind are NOT permitted.
• If you leave the exam room, you will not be allowed back in and your exam will be concluded. **Box your final answers for each question.**
• Start each numbered problem on a new page. All problems should be clearly numbered and in order.
• Leave your answers in terms of $\pi$ as necessary.

1. [20 points] Evaluate the following. These questions will be graded on your answer only. If an answer does not exist, write “DNE.”

   (a) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \frac{-\pi}{3}$  
   (b) $\sin^{-1}(\sin 2\pi) = 0$

   (c) $\cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$  
   (d) $\sin(\sin^{-1} 360) = \text{DNE}$

   (e) Rewrite $\cos(\tan^{-1} x)$ as an algebraic expression in $x$.  

   (f) Find the exact value of the expression: $\csc\left(\cos^{-1} \frac{8}{17}\right) = \frac{17}{15}$

For problems #2 - #5, show your work. Fully simplify all solutions. You must show a complete and valid solution method for full credit.

2. [20 points] The following questions are not related to each other.

   (a) Determine the following for the function $y = 4 \cos 3\pi t$.

      i. List the $t$-values of 4 consecutive zeros, ...$rac{8\pi}{3}$, $rac{14\pi}{3}$, $rac{20\pi}{3}$, ...$rac{32\pi}{3}$.

      ii. List the $t$-values of 3 consecutive maximums, ...$rac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, ...$rac{11\pi}{6}$.

      iii. List the $t$-values of 3 consecutive minimums, ...$rac{\pi}{3}$, $\frac{3\pi}{3}$, $\frac{5\pi}{3}$, ...$rac{13\pi}{3}$.

   (b) Find a trig function $f(x)$ for which all of the following are true:

      $\frac{dy}{dx} = 2\cos\left(\frac{1}{3}x\right)$ OR $\frac{dy}{dx} = 2\sin\left(\frac{1}{3}(x+\pi)\right)$

      • has 3 consecutive zeros at $x = 5\pi$, $7\pi$ and $9\pi$.
      • $f(10\pi) = -2$ is a local or relative minimum of the function, and
      • the domain of the function is all real numbers.

   OR: $\frac{dy}{dx} = -2\sin\left(\frac{1}{3}(x-\pi)\right)$ There are more possible answers.

   (c) Find an equation for the function shown to the right.

3. [20 points] A parcel of land is for sale. The parcel is in the shape of a triangle. All that is known about the parcel is:
side $a = 100$ yards, side $c = 160$ yards, and angle $A = 30^\circ$. Solve for all possible triangles that satisfy the given conditions.

   (a) For each possible triangle, draw the triangle showing the values of all angles and sides.

   (b) Find the area of each possible triangle.

Use these approximations as needed to obtain numerical values for your answers:

sin $23^\circ = 0.4$, sin $53^\circ = 0.8$, sin $97^\circ = 0.99$
4. [20 points] On a bicycle, the wheel sprocket has a radius of 2 inches and turns at a rate of $2\pi$ radians per second. A bike chain connects the wheel sprocket to a pedal sprocket as shown below. (Note that the figure below is not to scale.) For each question below, you must give the units with your answers.

(a) What is the linear speed of the bike chain? 

(b) The pedal sprocket is turning at a rate of $\frac{4\pi}{7}$ radians per second. What is the radius of the pedal sprocket? [Hint: Both sprockets have the same linear speed as the bike chain.]

(c) If the centers of the sprockets are 12 inches apart, how long is the bike chain? Use the figure to estimate the length of the chain.

5. [20 points] In the figure below, the angle $A$ and sides $b$ and $c$ in the triangle $ABC$ are known initially. Point $C$ is the center of the circle. (Note that the figure below is not to scale.)

(a) Find an equation for side $a$ in terms of sides $b$ and $c$ and angle $A$.

(b) Find an equation for angle $B$ and an equation for angle $C$ in terms of sides $a$, $b$ and $c$ and angle $A$. (Angles $B$ and $C$ are the angles inside triangle $ABC$.)

(c) The origin $(0, 0)$ for the figure above is at the point $B$. Given $A = 30^\circ$, $b = 3$ and $c = \sqrt{3}$, find the equation for the circle in standard form $(x-h)^2 + (y-k)^2 = r^2$. (Do not use these numerical values for questions (a) and (b), only for question (c).)

Extra credit. Graph the function $y = \sin^{-1} x$. Label everything.

WORTH 3%