1. [20 points] These questions will be graded on your answers only.

(a) Factor and simplify: \( y = \frac{5}{3} x^{2/3} - \frac{10}{3} x^{-1/3} \). Give your final expression without negative exponents.

(b) Express \( y \) as a function of \( x \): \( \log y - 2 = \log(x + 1) - \log x \)

(c) Simplify: \( \log_a (\sqrt{a})^x \)

(d) Find the value of \( \log_3 4 \cdot \log_4 5 \cdot \log_5 9 \). \( \text{Hint:} \) Think common base.

(e) True or False: A polynomial of degree 4 can have exactly one maximum value and one minimum value.

(f) Which of the following graphs shows how the rational function \( q(x) = \frac{x^2(x - 1)}{x(x + 1)^2} \) behaves as \( x \) approaches 0?

For problems #2 - #5, show your work. Fully simplify all solutions. You must show a complete and valid solution method for full credit.

2. [20 points] A professor gets a fresh cup of coffee at 8 am. In the equation below, \( t \) represents the time in minutes since 8 am, and \( F \) represents the temperature of the coffee in °F:

\[ t = -20 \left[ \ln(F - 30) - \ln(140) \right] \]

(a) Find the initial temperature of the coffee.

(b) When will the coffee be 100 °F? Use \( \ln 0.5 = -0.7 \) to obtain an approximate numerical value for your answer.

(c) Solve the equation for the temperature \( F \) of the coffee.

(d) What temperature will the coffee approach as time goes on?
3. [25 points] Given the function \( r(x) = \frac{-x^4 - 2x^3 + 3x^2}{x^3 + x^2 - 8x - 12} \), answer the following.

If the answer to any question is “none,” state “None.” Answers left blank will be marked incorrect.

(a) i. Give the numerator in factored form.
ii. Give the denominator in factored form.
iii. Give \( r(x) \) in factored form.

(b) Find any and all asymptotes of \( r(x) \).

(c) Find any and all intercepts and holes of \( r(x) \). Give your intercepts and holes in terms of their \((x, y)\) coordinates.

(d) Sketch the graph of \( r(x) \). Your sketch should clearly show the asymptotes and intercepts.

(e) What is the domain of \( r(x) \)?

4. [20 points] A polynomial function \( P(x) \) has zeros at \( x = -5, x = c \) with a multiplicity of 2 and \( x = 2c \), where \( c > 0 \), and no other zeros. The leading coefficient of \( P(x) \) is \( \frac{1}{2} \).

(a) What is the degree (highest power) of \( P(x) \)?

(b) For what values of \( x \) is \( P(x) \leq 0 \)?

(c) If \( P(x) \) has a \( y \)-intercept at the point \((0, -40)\), what is \( c \)?

(d) Suppose function \( G(x) = P(x - 6) \). What are the zeros of \( G(x) \)?

5. [15 points] A rumor that the dining hall will be serving liver and lima beans for dinner spreads through a college dorm of 270 undergrads according to the function

\[
L(t) = \frac{270}{1 + 8 \cdot 2^{-t}}
\]

where \( L \) is the number of students who have heard the rumor at time \( t \) in hours since it was first communicated.

(a) How many students started the rumor (at time \( t = 0 \))?  

(b) How many people know the rumor after 4 hours?

(c) At what time will half of the dorm know of the rumor?

Extra credit. At what time will 256 of the students in the dorm know of the rumor? Use \( \log_2 7 = 2.8 \) to get an approximate numerical value for your answer.

END OF EXAM