AT THE TOP OF THE PAGE write your name and your section number. Textbooks, class notes and electronic devices of any kind are NOT permitted. If you leave the exam room, you will not be allowed back in and your examination will be concluded.

**Do all your work on this exam. Only work done on this exam will be graded.**

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**Part I. Show Your Work.** Fully simplify all solutions. For these problems you must show a complete and valid solution method for full credit.

1. [20 points] The quadratic expression $x^2 - 3x + 1$ is a factor of the polynomial $P(x) = x^4 - 4x^3 - 16x^2 + 59x - 20$.

   (a) Find all the zeros of $P(x)$.

   \[
   \begin{align*}
   x^4 - 4x^3 - 16x^2 + 59x - 20 &= 0 \\
   (x^2 - 3x + 1)(x^2 - x - 20) &= 0 \\
   (x^2 - 3x + 1)(x - 5)(x + 4) &= 0 \\
   x &= 5, -4 \quad \text{or} \\
   x &= -1 \pm \sqrt{5}
   \end{align*}
   \]

   Answer (a): $-4, 5, -1 \pm \sqrt{5}$

   (b) Write $P(x)$ in fully factored form (all linear factors).

   \[
   P(x) = (x + 4)(x - 5)(x - \frac{3}{2} - \frac{\sqrt{5}}{2})(x - \frac{3}{2} + \frac{\sqrt{5}}{2})
   \]

   Answer (b): $\left( x - \frac{3}{2} - \frac{\sqrt{5}}{2} \right) \left( x - \frac{3}{2} + \frac{\sqrt{5}}{2} \right) \left( x - 5 \right) \left( x + 4 \right)$

   (c) How many relative (local) maximums does $P(x)$ have? 1

   How many relative (local) minimums does $P(x)$ have? 2

   (d) Complete each statement: As $x \to +\infty$, $y \to \infty$. As $x \to -\infty$, $y \to \infty$. 

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2. [15 points] (a) Find all the solutions to the equation \( \left( \frac{1}{5} \right)^{-2x+3} + 3 = 53. \)

\[
Z(\frac{1}{5})^{-2x+3} = 50
\]
\[
(5^{-1})^{-2x+3} = 25
\]
\[
5^{2x-3} = 5^2
\]
\[
2x-3 = 2
\]
\[
x = 5/2
\]

Answer (a): \( x = \frac{5}{2} \)

(b) Find all the solutions to the equation \( e^{2x} - e^x - 2 = 0. \)

\[
u = e^x \quad u^2 - u - 2 = 0
\]
\[
(u - 2)(u + 1) = 0
\]
\[
\rightarrow u = -1 \rightarrow e^x = -1 \rightarrow \text{No solution}
\]
\[
\rightarrow u = 2 \rightarrow e^x = 2 \rightarrow x = \ln 2
\]

Answer (b): \( x = \ln 2 \)

(c) Find all the solutions to the equation \( \log 100 + \log_4 32^2 - 2 \log_9 81 + \ln \sqrt{e} - 2 \log_2 3 - 1 = \log(x - 1) \)

\[
\log 100 + 2 \log_4 32 - 2 \log_9 81 + \ln e^{1/2} - 2 \log_2 3 - 1 = \ldots
\]
\[
2 + 2 \frac{\log 32}{\log 4} - 2Z + \frac{1}{2} - \frac{2 \log_2 3}{2} = \ldots
\]
\[
2 + 2 \frac{\log 32}{\log 2} - 4 + \frac{1}{2} - \frac{3}{2} = \ldots
\]
\[
2 + 2 \left( \frac{3}{2} \right) - 4 + \frac{1}{2} - \frac{3}{2} = 2 = \log (x-1)
\]
\[
10^Z = x-1
\]
\[
x = 10^Z + 1
\]

Answer (c): \( x = 101 \)
3. [15 points] Give the equation of a rational function that satisfies each of the following conditions.

(a) \( R(x) \) has a horizontal asymptote of \( y = 0 \) and a vertical asymptote of \( x = 2 \).

\[ R(x) = \frac{C}{x-2} \quad C \neq 0 \]

(b) \( S(x) \) has no \( x \)-intercepts, a horizontal asymptote of \( y = 3 \) and vertical asymptotes at \( x = 0 \) and \( x = 4 \).

\[ S(x) = \frac{3x^2+C}{x(x-4)} \quad C > 0 \]

(c) \( T(x) \) has a slant asymptote of \( y = x - 4 \) and vertical asymptote of \( x = 2 \).

\[ T(x) = \frac{x^2-6x+8+C}{x-2} \quad C \neq 0 \]

\( \text{Note: } \frac{x^2-6x+8}{x-2} = \frac{(x-4)(x-2)}{x-2} \)

\( \text{produces a deleted pt., not a slant asymptote} \)
4. [20 points] The radioactive isotope Precalcium-1235 (not a real element) is decaying in a lab. At 10 am there is 32g of the substance. One hour later there is 8g of the substance. Be sure to include the correct units with your answers.

(a) Find an exponential function the describes the amount of material \( m \) as a function of time \( t \) in hours from 10 am. Call this function \( m(t) \).

\[
m(t) = 32 \left( \frac{1}{2} \right)^{\frac{t}{1}} = 32 \left( \frac{1}{4} \right)^t
\]

Answer (a): 

(b) How much material is present at 12 noon?

\[ \frac{2}{g} \]

How much material was present at 9 am?

\[ 128 \text{ g} \]

How much material is present at 10:30 am?

\[ 16 \text{ g} \]

(c) When will there be 3.2g of Precalcium remaining? Use the table of values provided below to obtain an approximate numerical value for your answer.

\[ \frac{3.2}{32} = 0.1 = \left( \frac{1}{2} \right)^{\frac{t}{1}} \]

\[ \log 0.1 = 2 \log \left( \frac{1}{2} \right) \]

\[ + 1 = 2 \log [1 + \log 2] \]

\[ t = \frac{1}{2 \log 2} = \frac{1}{2(0.3)} = \frac{1}{0.6} = \frac{1}{0.6} \text{ hrs} = \frac{5}{3} = 1 \frac{2}{3} \text{ hrs} \]

Answer (c): 1 hr 40 min or 11:40 am

(d) When will there be 0 g of Precalcium remaining?

Answer (d): Never.

Table of Values of base-10 logs

<table>
<thead>
<tr>
<th>\log x</th>
<th>\log y</th>
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<tbody>
<tr>
<td>2</td>
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<tr>
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<td>4</td>
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<tr>
<td>8</td>
<td>0.90</td>
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<tr>
<td>9</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Part II. True/False, Short Answer [30 points total] These problems will be graded on your answers only.

5. True or False (CIRCLE your answer):
   a. A rational function can cross its horizontal asymptote but can never cross its vertical asymptote(s).
      True [ ] False [ ]
   b. $8 \cdot 2^x = 2^{3x}$
      True [ ] False [ ]
   c. $2^{3.5} = 8\sqrt{2}$
      True [ ] False [ ]
   d. $\frac{\log A}{\log B} = \log A - \log B$
      True [ ] False [ ]
   e. $\frac{1}{2} \log(x - 1) = \log \sqrt{x - 1}$
      True [ ] False [ ]
   f. $\frac{\log A}{\log B} = \frac{\ln A}{\ln B}$
      True [ ] False [ ]

6. If $a + bi$ is a zero of a polynomial $P(x)$, which of the following statements must be necessarily true (that is, must be true for all $a + bi$)? CIRCLE the statements that must be necessarily true.
   a. $P(a + bi) = 0$ [ ]
   b. $P(a - bi) = 0$ [ ]
   c. $P(0) = a + bi$ [ ]
   d. $a + bi$ is an $x$-intercept of the graph of $P(x)$ [ ]
   e. $(x - (a + bi))$ is a factor of the polynomial $P(x)$ [ ]

7. Give the following information about the function $y = \log(x + 1)$. Use interval notation where appropriate.
   a. domain: $(-1, \infty)$
   b. range: $(-\infty, \infty)$
   c. asymptotes: $x = -1$
   d. intercepts: $(0, 0)$

END OF EXAM