Part I. Show your work. Fully simplify all solutions. For these problems you must show a complete and valid solution method for full credit. Leave your answers in terms of $\pi$ as necessary.

1. (4 points) Simplify $\log_{16} \frac{1}{2} + \log_{16} 8 = \log_{16} \left( \frac{\frac{1}{2}}{8} \right) = \log_{16} \left( \frac{1}{4} \right)$

   1) Answer: $\frac{1}{2}$

2. (4 points) Solve for $x$: $x^3 + 6x^2 + 5 = -9x + 5$

   $x^3 + 6x^2 + 9x = 0$
   $x(x^2 + 6x + 9) = 0$
   $x(x + 3)^2 = 0$

   2) Answer: $x = 0, -3$

3. (4 points) Simplify $\cos^4 \theta + \sin^2 \theta \cos^2 \theta$.

   $\cos^4 \theta \left( \frac{\cos^2 \theta + \sin^2 \theta}{1} \right)$

   3) Answer: $\cos^3 \theta$

4. (4 points) Solve $5x^2 + x = 25^3$

   $5x^2 + x = 5^6$
   $x^2 + x = 6^6$
   $(x + 3)(x - 3) = 0$

   4) Answer: $x = -3, 3$

5. (4 points) Solve for $x$: $x^2 + 7x - 5x - 35 \geq 0$

   $x^2 + 2x - 35 \geq 0$
   $(x + 7)(x - 5) \geq 0$

   5) Answer: $(-\infty, -7] \cup [5, \infty)$
6. (4 points) Find all real numbers in the interval $[0, 2\pi)$, that satisfy the equation. \( \cos(2\theta) - \frac{1}{2} = 0 \)

\[
\cos(2\theta) = \frac{1}{2}
\]

\[2\theta = \frac{\pi}{3} + 2\pi n \Rightarrow \theta = \frac{\pi}{6} + \pi n \]

\[2\theta = \frac{5\pi}{3} + 2\pi n \Rightarrow \theta = \frac{5\pi}{6} + \pi n \]

6) Answer: \( \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6} \)

7. (4 points) Find all zeros (real and complex) of \( h(x) = x^3 + x^2 + x + 1 \)

\[
(x^2 + 1)(x + 1) = 0
\]

7) Answer: \( x = \pm i, -1 \)

8. (4 points) The growth in height of trees is frequently described by a logistic equation. Suppose the height \( h \) (in feet) of a tree at age \( t \) (in years) is

\[
h = \frac{120}{1 + 200e^{-0.2t}}.
\]

a. Based on this model, what is the maximum height trees may achieve?

b. Solve for \( t \).

8a. Answer: \( -\frac{\ln(\frac{120-h}{200h})}{0.2} \)

8b. Answer: \( t = -\frac{\ln(\frac{120-h}{200h})}{0.2} \)

9. (4 points) Evaluate \( \arcsin \left( \sin \frac{5\pi}{4} \right) \)

\[
\arcsin \left( -\frac{\sqrt{2}}{2} \right) = -\frac{\pi}{4}
\]

9) Answer: \( -\frac{\pi}{4} \)

10. (4 points) Solve for \( x \): \( \log^2 x - 3\log x - 28 = 0 \)

\[
(\log x + 4)(\log x - 7) = 0
\]

\[
\log x = -4, \log x = 7
\]

10) Answer: \( x = 10^{-4}, 10^7 \)
Part II Graphing. [n points each] For these problems you must show a complete and valid solution method for full credit. Leave your answers in terms of $\pi$ as necessary.

11. (10 points) December is historically one of the coldest months in Boulder, CO. The average daily low temperature is about $22^\circ$ F and the average daily high temperature is about $50^\circ$F. The temperature is typically halfway between the daily high and low temperature at both 10 a.m. and 10 p.m. and the highest temperatures are in the afternoon. Let $T(t)$ = the temperature in Boulder $t$ hours after midnight. Assume that the temperature is modeled as a trigonometric function (in terms of sine). See graph below. Find the (a) amplitude, (b) period, (c) phase shift, (d) vertical shift, and (e) the function $T(t)$.

![Graph of $T(t)$ function](image)

a. Amplitude: $\frac{14}{24}$
b. Period: $\frac{24}{36}$
c. Phase Shift: $\frac{10}{30}$
d. Vertical Shift: $\frac{36}{30}$
e. $T(t) = 14 \sin \left[ \frac{\pi}{10} (t-10) \right] + 36$

12. (8 points) Graph the following quadratic equation: $y = 3x^2 - 6x$. Make sure to label the vertex and any x-intercepts.

$y = 3x(x-2)$

x-intercept $(0,0)$ and $(2,0)$

Vertex $(1,-3)$

[-b/2a = 1]
13. (6 points) Find equations for the upper half and the lower half of the circle. \((x - 1)^2 + (y - 9)^2 = 1\)

14a) Upper half: \(y = \frac{\sqrt{1 - (x-1)^2}}{2} + 9\)
14b) Lower half: \(y = -\frac{\sqrt{1 - (x-1)^2}}{2} + 9\)

14. (12 points) Match each function with its graph. Write the letter in the box next to its graph. The graphs are not necessarily to the same scale.

A. \(f(x) = \frac{x^2}{x-2}\)
B. \(f(x) = \frac{x^2 - 3x + 2}{x^2 - x^2}\)
C. \(f(x) = x(x-1)(10x + 20)\)
D. \(f(x) = -(2-x)(x+1)(x-3)\)
E. \(f(x) = x(x^2 + 1)(x-2)\)
F. \(f(x) = \frac{x^2}{x-2}\)
Part III Short Answer, Multiple Choice, True/False. These problems will be graded on your answer only.

15. (6 points) A vinyl record spins at a rate of 1 revolution every 3 seconds. The vinyl has a radius of 4 inches. Give the linear speed of a bug on the edge of the record in ft/sec.

\[
\frac{1\text{ rev}}{3\text{ sec}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} \cdot \frac{1\text{ ft}}{1\text{ in}}
\]

Answer: \( \frac{2\pi}{9} \text{ ft/sec} \)

16. (6 points) Find the domain of the function \( h(x) = \sqrt{x - 6} - \log_6(9 - x) \). Use interval notation.

\[
\begin{align*}
x - 6 & \geq 0 \\
9 - x & > 0 \\
x & \geq 6 \\
9 & > x
\end{align*}
\]

Answer: \([6, 9)\)

17. (3 points) True or false: A function can cross an oblique asymptote. Answer: True False

18. (3 points) True or False: \( \log 500 + \log 200 = \log 700 \) Answer: True False

19. (3 points) True or False: \( \sin \left(2 \sin^{-1}(-\frac{1}{2})\right) = -\frac{\sqrt{3}}{2} \) Answer: True False

20. (3 points) True or False: \( \log \left(\frac{x}{y}\right) = \frac{\log x}{\log y} \) Answer: True False
Useful Formulas

\[
\begin{align*}
\sin(2\theta) &= 2\sin(\theta)\cos(\theta) \\
\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\
\cos(2\theta) &= 1 - 2\sin^2(\theta) \\
\cos(2\theta) &= 2\cos^2(\theta) - 1
\end{align*}
\]

\[
\begin{align*}
\sin \left( \frac{\theta}{2} \right) &= \pm \sqrt{\frac{1 - \cos(\theta)}{2}} \\
\cos \left( \frac{\theta}{2} \right) &= \pm \sqrt{\frac{1 + \cos(\theta)}{2}}
\end{align*}
\]

Difference quotient \[
\frac{f(x+h) - f(x)}{h}
\]

Equation of a circle: \((x - h)^2 + (y - k)^2 = r^2\)

Linear speed \(L = r\omega\), where \(r\) is radius and \(\omega\) is angular speed.

Arc length: \(s = r\theta\)

Sector area \(A = \frac{1}{2}r^2\theta\)