NAME: ____________________________________________

Instructor Name: ____________________________

Section: ___________

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write your Name, your Instructor’s name, and your Section number above, write your final answer in the BOXES or on the LINE provided.

1. (12 points) Consider the polynomial \( p(x) = x^4 + x^3 + 3x^2 + 5x - 10 \). Given that \( x^2 + 5 \) is a factor of this polynomial, answer the following:

(a) Write \( p(x) \) as a product of linear factors. 
\[
p(x) = (x + 2)(x - 1)(x - \sqrt{5}i)(x + \sqrt{5}i)
\]

\[
x^2 + x - 2
\]
\[
x^2 + 0x + 5) = x^4 + x^3 + 3x^2 + 5x - 10
\]
\[
-(x^4 + 0x^3 + 5x^2)
\]
\[
x^3 - 2x^2 + 5x
\]
\[
-(x^3 + 0x^2 + 5x)
\]
\[
-2x^2 - 10
\]
\[
-(-2x^2 - 10)
\]
\[
0
\]

(b) Give all zeros of the polynomial. \([-2, 1, \sqrt{5}i, -\sqrt{5}i]\)

(c) Give all x-intercepts of the polynomial. \((-2, 0), (1, 0)\)

2. (12 points) Given \( f(x) = \frac{4x - 2}{x + 3} \)

(a) Find \( f^{-1}(-3) \).
\[
x = \frac{4y - 2}{y + 3} \implies x(y + 3) = 4y - 2 \implies xy - 4y = -3x - 2
\]
\[
y = \frac{-3x - 2}{x - 4}
\]

Therefore, \( f^{-1}(-3) = \frac{-3(-3) - 2}{-3 - 4} = -1 \)

(b) Find the range of \( f^{-1} \). Domain of \( f = \) Range of \( f^{-1} = (-\infty, -3) \cup (-3, \infty) \)
3. (15 points) Consider the rational function, \( f(x) = \frac{(x^2 + x + 1)(x - 1)}{x^2 - 1} \)

(a) Give any asymptotes. If there aren’t any, state NONE.

asymptotes: slant asymptote: \( y = x \), vertical asymptote: \( x = -1 \)

\[
f(x) = \frac{(x^2 + x + 1)(x - 1)}{(x + 1)(x - 1)} = \frac{x^2 + x + 1}{x + 1}
\]

So there must be a vertical asymptote at \( x = -1 \) since this causes division by 0 and we’ve already eliminated the common factor \((x - 1)\).

To find the slant asymptote, we use polynomial division:

\[
\begin{array}{c|cc}
& x & x^2 + x + 1 \\
\hline
x + 1 & x^2 + x + 1 & -1 & 0 \\
& -(x^2 + x) & & \\
\hline
& 1 & &
\end{array}
\]

(b) Give any holes. If there aren’t any, state NONE.

hole: \( (1, \frac{3}{2}) \)

Since we cancelled a factor \((x - 1)\) in part \((a)\), There is a hole at \( x = 1 \). To find the y-coordinate, \( f(1) = \frac{1^2 + 1 + 1}{1 + 1} = \frac{3}{2} \)

(c) Give the x- and y-intercepts.

x-intercepts: \( x^2 + x + 1 = 0 \) has no real solutions...therefore no x-intercepts.
y-intercepts: Set \( x = 0 \), \( \implies y = 1 \), so the y-intercept is \((0, 1)\)

(d) Sketch a graph of the function below. Label the axes, the asymptotes if there are any, and the holes if there are any.
4. (12 points) TRUE or FALSE
(a) \( \frac{1}{x+y} = \frac{1}{x} + \frac{1}{y} \)  \[\text{FALSE}\]
(b) \( \left( \frac{2}{3} \right)^{-x} = \left( \frac{2}{3} \right)^{x} \)  \[\text{TRUE}\]
(c) It is possible to have a polynomial of degree 3 with roots \(-2, 1, 1 + 2i\).  \(\text{FALSE}\)
(d) \((x^4 + 16)^{1/4} = x + 2\) Assume \(x\) is positive.  \(\text{FALSE}\)
(e) The rational function \(r(x) = \frac{x^2 - 5x}{x^2 - x}\) has a vertical asymptote at \(x = 1\).  \(\text{TRUE}\)
(f) A rational function can never cross a vertical asymptote.  \(\text{TRUE}\)

5. (5 points) The polynomial function \(f(x) = -2x^4 + ax^3 + bx^2 + cx + d\) has zeros at \(-3, 3, 3, 6\). Which of the following could be the location of a relative maximum of the function? Circle all that apply.
(a) \(x = -4\)
(b) \(x = -1\)
(c) \(x = 3\)
(d) \(x = 4\)

6. (12 points) Give the equation of the polynomial based on the graph below. All of the x-intercepts are shown. The degree of the polynomial is 3. Give your answer in factored form. \(p(x) = -2x(x + 3)^2\)

\[p(x) = a(x + 3)^2(x), \implies 8 = a(-1 + 3)^2(-1), \implies 8 = a(4)(-1) \implies a = -2\]
7. (12 points) Let \( f(x) = \frac{x+3}{x-3} \) and \( g(x) = \frac{1}{x} \).

(a) Find and simplify \( f \circ g \)

\[
(f \circ g)(x) = \frac{\frac{1}{x} + 3}{1 - \frac{3}{x}} = \frac{1 + 3x}{x} \cdot \frac{x}{1 - 3x} = \frac{1 + 3x}{1 - 3x}
\]

So \( (f \circ g)(x) = \frac{1 + 3x}{1 - 3x} \)

(b) Find the domain. Give your answer in interval notation.

Since \( f(g(x)) = \frac{1 + 3x}{1 - 3x} \), \( 1 - 3x \neq 0 \implies x \neq 1/3 \)

Since the input function is \( g(x) = \frac{1}{x} \), \( x \neq 0 \). Therefore the domain is: \(( -\infty, 0 ) \cup ( 0, \frac{1}{3} ) \cup ( \frac{1}{3}, \infty )\)

8. (8 points) Fill in the blank. The function \( y = e^{x+1} + 2 \) has a

horizontal asymptote at \([ y = 2 ]\) and

y-intercept coordinate at \([ 0, e^1 + 2 ]\).

9. (12 points) A certain breed of mouse was introduced onto a small island with an initial population of 300 mice in 2004. Scientists estimate that the mouse population is doubling every year.

(a) Find a function \( N(t) \) of the form \( N(t) = N_0 \cdot a^t \) that models the number of mice after \( t \) years.

\[
N(t) = 300 \cdot 2^t
\]

at \( t = 0, N(0) = 300 \)

at \( t = 1, N(1) = 2 \cdot 300 \)

at \( t = 2, N(2) = 2 \cdot 2 \cdot 300 \)

at \( t = T, N(T) = 2 \cdot 2 \cdot \ldots 2 \cdot 300 = 2^T \cdot 300 \)

(b) Estimate the mouse population after 3 years. \( N(3) = 300 \cdot 2^3 = 2400 \) mice

(c) When will the mouse population be \( 1200\sqrt{2} \)? \( 2.5 \) years

\[
1200\sqrt{2} = 300 \cdot 2^t
\]

\[
4\sqrt{2} = 2^t
\]

\[
2^22^{1/2} = 2^t
\]

\[
\frac{5}{2} = t
\]