Textbooks, class notes and electronic devices of any kind are NOT permitted. Box or clearly indicate your final answers for each question.

Where it is applicable, give your final answers in radians, not degrees, and in terms of π.

Start each numbered problem on a new page. All problems should be clearly numbered and in order.

You may use this exam paper as scratch paper, but only work done in your bluebook will be graded.

Exam total points: 100

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For problems #1 - #3, show your work. Fully simplify all answers. You must show a complete and valid solution method for full credit.

1. [6, 6, 8, 4 points] The temperature of a car engine after it is turned off is modeled by the equation

$$\log\left(\frac{F - 20}{200}\right) = -0.4t$$

where $F$ is the temperature in degrees Fahrenheit and $t$ is the time in minutes after the engine is turned off. Give your answers with the correct units.

(a) At what time $t$ does the temperature of the engine reach 40 °F?

(b) What is the temperature of the engine when it is turned off?

(c) Solve the given equation for $F$ to obtain an equation for $F$ in terms of time $t$.

(d) What temperature does the engine approach as $t \to \infty$?

2. [3 @ 8 points each] These questions are not related.

(a) Find an equation of a sine function with period of $\pi$, an amplitude of 3 and a phase shift of $\pi/6$ to the right. Then list the first three consecutive positive zeros of the function.

(b) Find an equation of a tangent or cotangent function which passes through the points $\left(-\frac{\pi}{4}, -1\right)$ and $\left(\frac{\pi}{4}, 1\right)$ and has asymptotes at $x = ..., -2\pi, -\pi, 0, \pi, 2\pi, ...$.

(c) Find an equation for the cosine function in the graph below.

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(Exam continues on back.)
3. [3 @ 8 points each] These questions are not related. If a solution to any of these questions does not exist, state “DNE.”

(a) Find all solutions: \( 2^x + 2^{x+3} = 45 \).

(b) Find the average rate of change of the function \( f(x) = \sqrt{2} \cos\left(2x - \frac{\pi}{2}\right) \) from \( x = -\pi/8 \) to \( x = \pi/8 \), where average rate of change \( \text{ARC} = \frac{f(b) - f(a)}{b - a} \).

(c) Find the lengths of sides \( a \) and \( b \) shown in the figure to the right.

![Figure for #3 (c)](image)

4. [28 points total, 4 points each] These questions will be graded on your answers only. You do not need to show work. If an answer does not exist state “DNE.”

(a) A winch with a radius of 2 feet raises a large crate as it turns. If the winch turns at a rate of 15 rpm, how fast is the crate rising? Be sure to state your answer in whatever units you’ve chosen to use.

(b) The line in the figure to the right is \( 7x + 24y = 0 \). Find \( \tan \theta \).

(c) Evaluate \( \left( \cos \frac{2\pi}{3} + \sin \frac{7\pi}{4} \right)^2 \). Remember to fully simplify your answer.

(d) Find the exact value of the expression: \( \sec \left( \cot^{-1} 4 \right) \)

(e) Write the expression as an algebraic expression in \( x \) for \( x > 0 \): \( \sec \left( \sin^{-1} \frac{x}{\sqrt{x^2 + 64}} \right) \)

(f) What is the symmetry of the graph of \( y = \cos \left( x - \frac{\pi}{2} \right) \), odd, even or neither?

(g) Which one of the following most accurately describes the function \( y = \cos^{-1} x \)?

A. \( -\frac{\pi}{2} \leq \cos^{-1} x \leq \frac{\pi}{2} \)  
B. \( 0 \leq \cos^{-1} x \leq 2\pi \)  
C. \( -1 \leq \cos^{-1} x \leq 1 \)  
D. \( 0 \leq \cos^{-1} x \leq \pi \)  
E. \( -\infty \leq \cos^{-1} x \leq \infty \)

END OF EXAM