APPM 5430 HOMEWORK #3

Assigned Monday Sept. 30, 2019

DUE MONDAY OCT. 14, 2019

1. Define:

$$F(z) = \int_{-\infty}^{\infty} f(t)e^{-izt}dt$$

$F(z)$ is referred to as the Fourier transform of $f(t)$.

i) Let $f(t) = g(t)h(t)$ where $g(t)$ is a bounded continuous function for all t and $h(t) = \{e^{-\kappa_1 t}, t > 0, \quad e^{\kappa_2 t}, t < 0\}, \kappa_j > 0, j = 1, 2$. Find the region where $F(z)$ is analytic.

ii) Do the same if $f(t) = e^{-\kappa t^2}$. What can be said about the analyticity of $F(z)$?

2. Given the series:

$$F(z) = 1 + 2z + 3z^2 + \cdots = \sum_{n=1}^{\infty} nz^{n-1}$$

i) Find its radius of convergence ii) Find the analytic continuation of this function for all z.

3. Suppose we are given the 'exponential' integral function:

$$F(z) = \int_{-\infty}^{z} \frac{e^t}{t}dt$$

for $z = x + iy, \mathrm{Im}z < 0, x < 0$. Discuss the analytic properties and analytic continuation of this function to $\mathrm{Im}z > 0, x < 0$.

3.5. Answer the question of Problem 3.5.1 for the functions:

$$(a)\ \frac{z^2}{z^2 + 2z + 1} \quad (b)\ \cot \frac{2}{z^2}, \quad (c) e^{\cosh z}$$

3.6 4; 7a,b,c

PROBLEMS CONTINUE–NEXT PAGE

Additional HW problems:

1. Find a Mittag-Leffler expansion of a meromorphic function $f(z)$ which has as its only poles:

a) simple poles at $\sqrt{j},\ j = 1, 2, 3...$ with unit residues at these pole locations

b) simple poles at $\log(j),\ j = 2, 3, 4...$ with unit residues at these pole locations

2. Find a Weierstrass expansion of an entire function $f(z)$ which has as its only zero's:

a) simples zero's at $\sqrt{j},\ j = 1, 2, 3...$

b) simples zero's at $\log(j),\ j = 2, 3, 4...$