1. Define:

\[ F(z) = \int_{-\infty}^{\infty} f(t) e^{-izt} dt \]

\( F(z) \) is referred to as the Fourier transform of \( f(t) \).

i) Let \( f(t) = g(t)h(t) \) where \( g(t) \) is a bounded continuous function for all \( t \) and \( h(t) = \{ e^{-\kappa_1 t}, t > 0, \quad e^{\kappa_2 t}, t < 0 \} \), \( \kappa_j > 0, j = 1, 2 \). Find the region where \( F(z) \) is analytic.

ii) Do the same if \( f(t) = e^{-\kappa t^2} \). What can be said about the analyticity of \( F(z) \)?

2. Given the series:

\[ F(z) = 1 + 2z + 3z^2 + \cdots = \sum_{n=1}^{\infty} nz^{n-1} \]

i) Find its radius of convergence ii) Find the analytic continuation of this function for all \( z \).

3. Suppose we are given the ‘exponential’ integral function:

\[ F(z) = \int_{-\infty}^{z} \frac{e^t}{t} dt \]

for \( z = x + iy, \Im z < 0, x < 0 \). Discuss the analytic properties and analytic continuation of this function to \( \Im z > 0, x < 0 \).

3.5. Answer the question of Problem 3.5.1 for the functions:

(a) \( \frac{z^2}{z^2 + 2z + 1} \)  (b) \( \cot \frac{2}{z^2} \)  (c) \( e^{\cosh z} \)

3.6 4; 7a,b,c

PROBLEMS CONTINUE–NEXT PAGE
Additional HW problems:

1. Find a Mittag-Leffler expansion of a meromorphic function $f(z)$ which has as its only poles:
   a) simple poles at $\sqrt{j}$, $j = 1, 2, 3...$ with unit residues at these pole locations
   b) simple poles at $\log(j)$, $j = 2, 3, 4...$ with unit residues at these pole locations

2. Find a Weierstrass expansion of an entire function $f(z)$ which has as its only zero’s:
   a) simples zero’s at $\sqrt{j}$, $j = 1, 2, 3...$
   b) simples zero’s at $\log(j)$, $j = 2, 3, 4...$