APPM 4360/5360 Introduction to Complex Variables and Applications

HOMEWORK #1

Assigned: Monday Jan. 14, 2019

DUE: In class Monday January 28, 2019

1. Express each of the following complex numbers in polar exponential form: $re^{i\theta}$

a. $-2i$; b. $\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$; c. $\sqrt{3} - i$;

2. Express the following complex numbers in real: form $x + iy$, where x and y are real:

a. $\frac{1}{1-2i}$; b. $(1-i)^2(1+2i)$; c. $|1-3i|$

3. Solve for all the roots of the following equation: $z^3 - 2z^2 + 2z = 0$

4. Establish the following inequalities:

a. $|4z_1 - z_2| \leq 4(|z_1| + |z_2|)$; b. $|2z_1\bar{z}_2 + 3\bar{z}_1 z_2| \leq 5|z_1||z_2|$

5. Sketch the region associated with the following inequality and determine if the region is open, closed, bounded, compact, connected: $6 \leq |3z + 7| \leq 9$; Explain.

6. Show that $\mathrm{Im}(\frac{1}{z})$ and $\mathrm{Im}(-z)$ have the same sign for all $z \neq 0$.

7. Find the series expansion around $z = 0$ of: $\frac{\sin z - z}{z^2}$

8. Evaluate the following limits, explain reasoning:

a. $\lim_{z\to 0} \frac{\cos\beta z - 1}{z^2}, \beta \neq 0$ const., b. $\lim_{z\to 0} \frac{\sin\alpha z}{\sin\beta z}, \alpha, \beta \neq 0$ const.

c. $\lim_{z\to\infty} \frac{Mz^4+z}{(Nz^2+3)^2}; M, N \neq 0$, const. d. $\lim_{z\to\infty} \frac{\sinh 2az}{\cosh 2az}, a > 0$ const.

9. a) Solve 1.3: 3

b) Where are the following functions differentiable: i) $\tanh z$ ii) $e^{1/(z-i)}$

10. Find the general solution of the following differential equation: $\frac{d^3w}{dz^3} - 8w = 0$; write the solution in **real** form.

XC (Extra Credit) Use 'ϵ, δ' formulation to **prove** that $\lim_{z\to i} z^2 = -1$