University of Colorado at Boulder
Department of Applied Mathematics

Comprehensive Examination

Friday, April 25, 2014
10:00 a.m.
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Presenter

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Title

Polyharmonic Splines with Polynomials for the Numerical Solution of Convective PDEs

Conventional radial basis function (RBF) methods for PDEs offer geometric flexibility and high orders of accuracy, with no need for a mesh [1]. Derivative approximations are based only on function values at scattered node locations. The more recently developed RBF-generated finite differences (RBF-FD) approach is computationally much faster, since all derivative approximations are local. However, a direct application of RBF-FD eventually suffers from saturation errors (failure of convergence under refinement), and has been restricted to small stencil-sizes for problems with boundaries [3].

We have found that RBF-FD can be improved dramatically by augmenting the RBF basis with polynomials up to some fixed degree, eliminating saturation errors. Furthermore, since RBFs are still part of the basis, the results are accurate even at low resolution. In addition to modifying the local basis, fictitious points or ghost nodes can be placed just outside boundaries to lessen the effects of the Runge phenomenon and allow for larger stencil sizes. This ghost node approach is similar to what has been used previously in conjunction with conventional finite-differences, but the RBF-based method is not restricted to Cartesian grids. Finally, with polynomials included in the basis, simple finitely-differentiable polyharmonic spline (PHS) RBFs can be used instead of the traditional smooth ones, such as Gaussians or Multiquadrics. Using PHS RBFs, derivatives can be approximated on any reasonable set of nodes, regardless of the average node-spacing, without the need to choose a shape parameter.

To demonstrate the modified RBF-FD method, it will first be applied to the simple acoustic wave equation on the unit disk, where the exact solution is known analytically, and numerical solutions can be seen to converge. For further demonstration, we will apply the method to some test cases from numerical weather prediction, where the exact solution is unknown and convergence is much more difficult to verify, or in some cases (with turbulence) out of the question [2].

With ghost nodes and a modified interpolation basis, RBF-FD competes well with existing methods not only for toy problems, but also for important atmospheric test cases. Eventually, RBF-FD could be used with appropriate weather parameterizations to form a full 3D model for numerical weather prediction.

References

[1] Natasha Flyer and Grady Wright, *Transport schemes on a sphere using radial basis functions*, J. Comput. Phys. 2007

[2] F.X. Giraldo and M. Restelli, *A study of spectral element and discontinuous galerkin methods for the Navier-Stokes equations in nonhydrostatic mesoscale atmospheric modeling: Equation sets and test cases*, J. Comput. Phys. 2007

[3] H.Q. Chen and C. Shu, *An efficient implicit mesh-free method to solve two-dimensional compressible euler equations*, International Journal of Modern Physics C, 2005