Ninth Annual Front Range Applied Mathematics Student Conference

March 2, 2013

Registration: 8:30 - 9:00

Morning Session I - NC1311
9:00 - 11:35

9:00 - 9:20	Yi-Hung Kuo *University of Wyoming*	Analysis of Dispersion-Relation-Preserving Schemes
9:25 - 9:45	Jennifer Maple *Colorado State University*	Steady State Hopf Mode Interaction in Anisotropic System
9:50 - 10:10	Jason M. Gates *Colorado School of Mines*	Developing Adaptive Multiscale Finite Element Methods for Heterogeneous Media
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Victoria Li *University of Colorado, Boulder*	Mathematically Modeling Tsunamis
10:50 - 11:10	Prosper K. Torsu *University of Wyoming*	A Method of Perturbation for Solving Semilinear Elliptic Problems
11:15 - 11:35	John Villavert *University of Colorado, Boulder*	An Inviscid Regularization of Hyperbolic Conservation Laws

Morning Session II - Room NC1312
9:00 - 11:35

9:00 - 9:20	Matthew Niemerg *Colorado State University*	An Introduction to Parameter Homotopies
9:25 - 9:45	John Lepird *United States Air Force Academy*	Modeling Potential Airborne Bird Strike Countermeasures
9:50 - 10:10	Eric Hanson *Colorado State University*	Vector Bundle Based Maps and Metrics for Improving Topological Signals from Persistent Homology
10:10 - 10:25	15 Minute Break	

10:25 - 10:45	Jennifer Diemunsch *University of Colorado, Denver*	2-Factors with a Bounded Number of Odd Cycles
10:50 - 11:10	Sofya Chepushtanova *Colorado State University*	Comprehensive Analysis of Hyperspectral Data Using Band Selection Based on Sparse Support Vector Machines
11:15 - 11:35	Mark Mueller *University of Colorado, Denver*	The Physics of Extracting Coal Bed Methane

Morning Session III - Room NC1313
9:00 - 11:35

9:00 - 9:20	Ruth A. Martin *University of Colorado, Boulder*	Three-Wave Mixing and the Explosive Instability
9:25 - 9:45	Jewell Anne Hartman *University of Colorado, Colorado Springs*	Partial Differential Equations for a Statistical View of Biological Dynamics for the Interdisciplinary Studies of the Mathematical and Physical Biology of the Cell in the Advancement of Medicine
9:50 - 10:10	Quanling Deng *University of Wyoming*	Symmetric Interior Penalty Galerkin Method for Solving Semilinear Elliptic Problems
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Francis C. Motta *Colorado State University*	On Optimally Topologically Transitive Orbits
10:50 - 11:10	Bradley Lowery *University of Colorado, Denver*	Stability and Parallel Performance Analysis of Oblique QR Factorization Algorithms
11:15 - 11:35	Bradley McCaskill *University of Wyoming*	Using a Robin Boundary Condition within Domain Decomposition to Solve Elliptic Partial Differential Equations

Lunch: 11:45 - 12:25

Plenary Address: 12:30 - 1:30, NC1130

Dr. Loren Cobb, *University of Colorado, Denver*

Mathematics of Society and its Dysfunctions

Group Photographs at 1:30

Afternoon Session I - Room NC1311
1:45 - 3:50

1:45 - 2:05	Nathan Pelc *United States Air Force Academy*	*Modeling Contest, Problem A.* Cooking the Ultimate Brownie: Introduction to Three-Dimensional Finite Element Analysis
2:10 - 2:30	Danny Dauwe, Taylor Klotz, and Jessica Gronski *University of Colorado, Colorado Springs*	*Modeling Contest, Problem A.* The Ultimate Brownie: A Guide to Make the Ultimate Brownie Pan
2:35 - 2:55	Chuan Zhang, Gerhard Dangelmayr and Iuliana Oprea *Colorado State University*	Storing Cycles in Neural Networks with Delayed Coupling
3:00 - 3:20	Tim Marrinan *Colorado State University*	The Flag of Best Fit as a Representative for a Collection of Linear Subspaces
3:25 - 3:45	Christopher Aicher, Abigail Z. Jacobs and Aaron Clauset *University of Colorado, Boulder*	Weighted Stochastic Block Mode

Afternoon Session II - Room NC1312
1:45 - 3:50

1:45 - 2:05	Victoria Gershuny *University of Colorado, Boulder*	Autoregulation of Retinal Blood Flow in Patients with Glaucoma
2:10 - 2:30	Roberto Munoz-Alicea *Colorado State University*	HIV-1 Gag Trimerization Near Cell Membrane: Mathematical Models and Numerical Simulations
2:35 - 2:55	Casey Moffatt *University of Colorado, Denver*	A Look at the Potential Function for Multi-Graphs

3:00 - 3:20	Steven Ihde *Colorado State University*	Numerical Preconditioning for Homotopy Continuation
3:25 - 3:45	Timothy Morris *University of Colorado, Denver*	Cloud Conveyor System

Afternoon Session III - Room NC1313
1:45 - 3:20

1:45 - 2:05	Brandon A. Mueller *United States Air Force Academy*	Validating Falcon Telescope Network Simulations with Preliminary Data
2:10 - 2:30	Volodymyr Y. Kondratenko, Jonathan D. Beezley, Jan Mandel and Adam K. Kochanski *University of Colorado, Denver*	Assimilation of Fire Perimeter Data into the Fire Spread Model SFIRE Coupled with the WRF Model
2:35 - 2:55	Brent Davis *Colorado State University*	Application of Numerical Algebraic Geometry to Geometric Data Analysis
3:00 - 3:20	Lawrence Bush *University of Wyoming*	Conservative Flux from the Continuous Galerkin FEM
3:25 - 3:45	Ryan Hartz *United States Air Force Academy*	Testing the Effects of Altitude on Rocket Motor Performance

Plenary Speaker

12:30 - 1:30, NC1130

Mathematics of Society and its Dysfunctions

Dr. Loren Cobb

Department of Mathematical and Statistical Sciences and Department of Sociology
University of Colorado, Denver

Social problems are not the traditional domain of applied mathematics, yet remarkable progress is being made, mostly by physicists. Even most mathematicians are unaware of the impending explosion of new ideas and new mathematics for the social sciences. As recently as 25 years ago this endeavor simply did not exist. Over the past 20 years I have worked as a free-lance applied mathematician with a dozen different countries on their toughest social problems: economic (poverty, underground economies), criminal (drug trafficking, juvenile gangs), political (corruption, ethnic relations, reform of the police), and demographic (refugees, migration, population growth). In every case there has been wonderful mathematics to be discovered, and nontrivial questions of stability and change to ponder. This is a field in its infancy, where new ideas are welcomed like water for a man dying of thirst.

MORNING SESSION I

ANALYSIS OF DISPERSION-RELATION-PRESERVING SCHEMES

Yi-Hung Kuo
University of Wyoming

A numerical scheme which has the same dispersion relation as the original partial differential equation is referred to as a dispersion-relation-preserving (DRP) scheme. The principle of constructing a DRP scheme is to optimize the approximations of the space and time derivatives in the wave-number and Fourier spaces. DRP schemes are favorable for large-scale wave propagation problems, for which a coarse mesh may be preferable due to limitations of computer power. In this talk I will review the basic concepts of DRP schemes. The advantages as well as the limitations of DRP schemes will be revealed from both the theoretical and practical aspects. This work is joint with Long Lee and Gregory Lyng at the University of Wyoming.

STEADY STATE HOPF MODE INTERACTION IN ANISOTROPIC SYSTEM

Jennifer Maple
Colorado State University

We present a theoretical and numerical analysis of a system of four globally coupled complex Ginzburg-Landau equations modeling steady oblique-normal Hopf mode interaction observed in experiments in electroconvection of nematic liquid crystals. Numerical simulations of the GC-CGL equations for parameters corresponding to the mode interaction show a variety of patterns, including steady oblique rolls, normal traveling rolls, and mixed modes.

DEVELOPING ADAPTIVE MULTISCALE FINITE ELEMENT METHODS FOR HETEROGENEOUS MEDIA

Jason M. Gates
Colorado School of Mines

This work is concerned with developing efficient methods to simulate the classical uniformly elliptic partial differential equation (PDE) with heterogeneous coefficients. When the coefficient function is well-behaved, the standard Galerkin Finite Element Method (FEM) produces excellent results. However, in cases with problematic coefficients (for instance, highly oscillatory, has high contrast, or both) the standard Galerkin FEM is prohibitively expensive to capture the multiscale phenomena present in the problem, even with adaptive mesh refinement techniques. Such high-contrast problems arise, for instance, when modeling sub-surface fluid flow. As such, numerical methods that can adequately resolve the multiscale features of the solution with computational effort dominated only by solving several local problems in parallel(and the global problem solved only on a coarse-grid) are very desirable in the petroleum industry and elsewhere. This research is concerned with developing such a new class of adaptive Multiscale Finite Element Method (MsFEM).

A one-dimensional code has been developed and validated to handle highly oscillatory and high-contrast problems using known MsFEM techniques. A two-dimensional code is currently being developed. At the moment, the 2-D code is implemented by developing nodal multiscale (partition of unity) nonlinear multiscale basis functions by using the appropriate linear boundary conditions for local problems. This will be used in a future work as an initial approximation to iteratively and adaptively generate accurate multiscale basis functions. Current work involves implementing the so-called oscillatory boundary conditions that can better reflect the characteristics of the heterogeneous medium, leading to better initial approximation of our adaptive MsFEM. Future work will involve an iterative improvement on the sub-grid boundary conditions to adaptively resolve multiscale features (such as those described by random fields) as well as possible extensions to time-dependent problems and/or three dimensions.

MATHEMATICALLY MODELING TSUNAMIS

Victoria Li
University of Colorado, Boulder

Theoretically, tsunamis are described by a system of nonlinear hyperbolic partial differential equations (PDEs) called the Shallow Water Equations. We use the method of characteristics and shock theory to solve for the propagation of the wave numerically, given a set of initial conditions. This is part of a larger project to develop an accurate tsunami forecast model, a collaboration between Professor Segur, the National Oceanic and Atmospheric Administration (NOAA), and other researchers. In the past, the Korteweg-deVries equation successfully described shock waves called solitons, but the shock waves produced by tsunamis are different and require a new model, such as the model proposed by Serre, Su-Gardner, and Green-Naghdi. Our goal is to determine whether the method of characteristics can generate results which match actual data, and if not, how the model can be improved.

A METHOD OF PERTURBATION FOR SOLVING SEMILINEAR ELLIPTIC PROBLEMS

Prosper K. Torsu
University of Wyoming

In this talk, we present a method of perturbation for solving elliptic PDEs. The method relies on representing the elliptic coefficient as a truncated Taylor series expansion. With this in place, the approximate solution is expressed as a truncated series whose n-th order term is governed by a linear elliptic problem that depends on the lower order terms for n greater than one. The only term in the series that is still governed by a semilinear elliptic equation is the zeroth order term. Each of the governing equations is obtained by performing an "order-matching" of the differential operators and the force functions. The solution of the governing equations are obtained sequentially. We establish numerical bounds for the gradient of each solution term, which indicate the stability of the solution. Numerical experiments also indicates the convergence of the true solution. One can see the potential viability of the proposed method: we only need to solve one nonlinear zeroth order equation (with elliptic coefficient 1) and linear equations for the higher order terms (also with elliptic coefficient 1). This is especially pronounced for Monte Carlo type simulations.

AN INVISCID REGULARIZATION OF HYPERBOLIC CONSERVATION LAWS

John Villavert
University of Colorado, Boulder

This talk introduces a technique that utilizes a filtering or spatial averaging of the nonlinear terms in the hyperbolic equations as an inviscid regularization of shocks. A central motivation for the analytical study we present here is to promote a recently developed filtering technique, the observable divergence method, rather than viscous regularization, as an alternative to the simulation of shocks and turbulence for compressible flows while, on the other hand, generalizing and unifying previous mathematical and numerical analysis of the method applied to the one dimensional Burgers' and Euler equations. We will primarily address two fundamental issues on the mathematical analysis of this filtering technique. The first is on the global-in-time existence and uniqueness of classical solutions for the regularization under the more general setting of quasilinear, symmetric hyperbolic systems in higher dimensions. The second issue examines one dimensional scalar conservation laws and describes the sufficient conditions that guarantee the compactness property needed in showing the technique captures the unique physical solution of the original problem as filtering vanishes.

MORNING SESSION II

AN INTRODUCTION TO PARAMETER HOMOTOPIES

Matthew Niemerg
Collaborators: Daniel J. Bates and Daniel Brake
Colorado State University

Numerical algebraic geometry provides a number of efficient tools for approximating the solutions of polynomial systems. One such tool is the parameter homotopy, which allows for the rapid computations of the solutions of a polynomial system if the set of isolated, nonsingular solutions for a problem with the same monomial structure but different coefficients has previously been computed. Parameter homotopies have recently been used in several areas of application and have been implemented in at least two software packages. The talk will describe a new, parallel, optimized implementation, Paramotopy, of this technique, making use of the implementation in the Bertini software package. The novel features of this implementation not available elsewhere are that it allows for the simultaneous solutions of arbitrary polynomial systems in a parameterized family on an automatically generated (or manually provided) mesh in the parameter space of coefficients, includes front ends and back ends that are easily specified to particular types of problems, and provides adaptive techniques for solving polynomial systems near singular points in the parameter space.

MODELING POTENTIAL AIRBORNE BIRD STRIKE COUNTERMEASURE

John Lepird
Collaborator: David Tipton
United States Air Force Academy

Contrary to popular belief, aircraft/bird collisions are not just a problem for the birds: every year, bird strikes result in over 32 million in damages and an average of 1.3 fatalities in the US Air Force alone. Recent population growth in large migratory birds, such as Canada geese, has exacerbated the problem. Currently, programs such as habitat management and falconry have been very effective in reducing the bird strike danger in the immediate vicinities of airports, but aircraft remain in danger during initial ascent and final decent. Consequently, an airborne bird deterrence system is necessary. An accidental discovery at MITs Lincoln Laboratory demonstrated promise for a speaker array. Before prototype fabrication, mathematical analysis was required in order to determine the feasibility of such a program. This research focuses on this analysis. The project goal was to simulate what a bird hears from an approaching aircraft, and use this to determine feasibility of an acoustical bird strike countermeasure. A number of physical phenomena including Doppler shift, frequency-specific atmospheric attenuation, and orientation dependency made the problem both complex and interesting. Various numerical analysis techniques such as the secant method, finite difference derivative approximation, and composite Simpsons integration were critical in solving this complex problem. Combining these numerical methods with data from the Department of Transportation allowed for an approximate solution in the general case: based on any bird and aircraft flight path, the model generates frequency domain, bird-perception weighted sound profiles at all points in time. This simulation demonstrates feasibility of an airborne acoustic bird strike countermeasure. Consequently, a working prototype is currently awaiting initial tests on live specimens.

VECTOR BUNDLE BASED MAPS AND METRICS FOR IMPROVING TOPOLOGICAL SIGNALS FROM PERSISTENT HOMOLOGY

Eric Hanson
Collaborators: Francis Motta, Chris Peterson, and Lori Ziegelmeier
Colorado State University

Capillarity is the physical phenomenon of fluid rise against a vertical wall. The height of the fluid at the wall interface depends upon the geometry of the wall, the wall material and the fluid, and is predicted by the Laplace-Young capillary equation. Of particular interest to industrial semiconductor dipping applications, for example, is the height rise due to wedges of various angles. However, the solution of the Laplace-Young capillary equation for small-angled wedges ($0 < \alpha < \frac{\pi}{2} - \gamma$, where γ is the angle of the contact against a flat wall - dependent upon material and fluid properties - and 2α is the wedge angle) is unbounded as it approaches the corner, and standard finite element methods fail to converge. We propose using the first-order system LL* (FOSLL*) finite element method to determine numerical solutions for the Laplace-Young capillary equations for the small-angle wedge.

2-FACTORS WITH A BOUNDED NUMBER OF ODD CYCLES

Jennifer Diemunsch, Michael Ferrara, Samantha Graffeo, and Timothy Morris
University of Colorado, Denver

A 2-factor in a graph is a collection of disjoint cycles which span the graph. This talk considers the number of odd cycles in a 2-factor, with specific interest in claw-free graphs. A 2-factor with no odd cycles is equivalent to a pair of disjoint perfect matchings, so we conclude with a discussion of 2-factors with no odd cycles.

COMPREHENSIVE ANALYSIS OF HYPERSPECTRAL DATA USING BAND SELECTION BASED ON SPARSE SUPPORT VECTOR MACHINES

Sofya Chepushtanova
Colorado State University

Hyperspectral data are rich in information. This results in several challenges while processing them, including relevant information identification and classification accuracy. In this research we explore an ℓ_1-norm (sparse) linear support vector machine (ℓ_1-norm SVM or SSVM) for the hyperspectral imagery band (wavelength range) selection problem. This approach allows one to classify data and simultaneously select relevant (to this particular classification task) bands.

An SSVM constructs the optimal separating hyperplane between two classes of data points with maximum margin measured in ℓ_∞-norm (this optimization problem can be written in the form of a linear program which we solve using a primal dual interior point path following method). The ℓ_1-norm suppresses many components of the weight vector, w (normal to the hyperplane), and the nonzero components in w indicate the spectral bands that are effective at separating the data. This gives us a basis for proposing a statistical framework for the band selection problem. To reduce variability in decision functions, we use bagging (bootstrap aggregating) based on building SSVMs on randomly chosen subsets of training data, and taking the average of the decision functions as a final SSVM model. After bagging, we have a sample of weight values for each band. Exploring each sample by hypothesis testing, we make a decision about redundancy of the band.

Since the SSVM is a binary method, we use one-against-one approach with majority voting to deal with multiple classes. Also, possible approaches to find the optimal subset of bands for all the classes are considered. The behavior of the learning method is illustrated by modeling sparse classifiers for the AVIRIS Indian Pines data set.

THE PHYSICS OF EXTRACTING COAL BED METHANE

Mark Mueller
University of Colorado, Denver

Coal bed methane is an abundant and clean-burning fuel, well suited to today's energy needs. The United States may have 10^{14} cubic feet of recoverable coal bed methane. Furthermore, up to 90% of all the coal in the US cannot be mined under current environmental regulation, so coal bed methane offers an attractive alternative.

This talk will discuss the physics of methane extraction from coal beds. The extraction process presents problems not previously encountered in the coal or oil industry, such as methane adsorption and diffusion, as well as stress-dependent mechanical properties of coal such as permeability.

MORNING SESSION III

THREE-WAVE MIXING AND THE EXPLOSIVE INSTABILITY

Ruth A. Martin
University of Colorado, Boulder

The resonant interaction of three waves is one of the simplest forms of nonlinear interaction for dispersive waves of small amplitude. This behavior arises frequently in applications ranging from nonlinear optics to internal waves through the study of the weakly nonlinear limit of a dispersive system. The slowly varying amplitudes of the three waves satisfy a set of coupled, nonlinear partial differential equations known as the three-wave equations. Furthermore, without spatial dependence, the PDEs reduce to a set of coupled ODEs that constitute a completely integrable Hamiltonian system. Solutions to the ODEs can exhibit two distinct behaviors, depending on the parameters of the problem. In the first case, solutions are bounded, while in the second case solutions blow up in finite time. The first case is referred to as the nonexplosive case, while the second case is known as the explosive instability. Solutions in the explosive case can be written in terms of a Laurent series with six free parameters.

By considering the ODEs in their Hamiltonian form, we are able to reduce the two distinct cases to a single case. The solution to the ODEs can be written in terms of elliptic functions, and the parameters of the elliptic function, along with the initial data, help determine whether we are in the explosive or the nonexplosive case. We also find that every nonexplosive case has a related explosive case, while the reverse is not true. Currently, the explosive case is better understood than the nonexplosive case. However, we can take advantage of the relationship between the two cases in order to gain information about the nonexplosive case. When spatial dependence is re-introduced, we hope to use our knowledge of the analytic structure of the ODEs, combined with the integrable structure of the PDEs, in order to solve the full three-wave equations in new configurations.

PARTIAL DIFFERENTIAL EQUATIONS FOR A STATISTICAL VIEW OF BIOLOGICAL DYNAMICS FOR THE INTERDISCIPLINARY STUDIES OF THE MATHEMATICAL AND PHYSICAL BIOLOGY OF THE CELL IN THE ADVANCEMENT OF MEDICINE

Jewell Anne Hartman
University of Colorado, Colorado Springs

An interdisciplinary frontier exists between mathematics, physics, biology, and medicine; this frontier is known as biophysics. To expand this frontier, sophisticated techniques and tools from the field of applied mathematics are necessary to benefit the field. In order to study biological dynamics, analyzing rates of biochemical processes is the first step in understanding the fundamentals of biophysics. One of the tools necessary involves the field of partial differential equations, both linear and nonlinear, and it is greatly significant to the field.

To study the statistical view of biological dynamics, it is necessary to understand diffusion. Ficks Law, the Diffusion Equation, and the Conservation of Mass will be used to study concentration fields and diffusive dynamics. Biological cells can be thought of as chemical factories, and it is a known fact that chemicals diffuse. Consequently, by studying partial differential equations, rate equations for polymerization reactions, dynamics of molecular motors, macromolecule decay, and enzyme kinetics are just a few of the fields of biophysics that can be gained from applied mathematics. In order to study these systems, applied mathematics is a necessary field. Statistical biological dynamics also involves electrostatics. Problems involving electrostatics require Maxwell Equations, which are some of the most significant differential equations that exist. Diffusion equations for biophysics require knowledge of partial differential equations in an applied sense, modeling the situation to study the specifics of the field. The ultimate goal of the UCCS BioFrontiers Institute Research Program is to expand research from an interdisciplinary program combining biology, computer science, mathematics, medicine, and physics. But, without applied mathematics, these advances could not be made. Therefore, in order to succeed in accomplishing the future of medicine as the frontier between physics and biology is discovered, applied mathematics will serve as the tool that makes it all possible.

SYMMETRIC INTERIOR PENALTY GALERKIN METHOD FOR SOLVING SEMILINEAR ELLIPTIC PROBLEMS

Quanling Deng
University of Wyoming

We consider numerical discretization of a class of semilinear elliptic problems using the Symmetric Interior Penalty Galerkin (SIPG). The resulting algebraic system is then linearized using Newton's method of iteration. Several numerical examples are presented to illustrate performance of the numerical procedure. Problems such as Bratus equation and nonlinear polynomial reaction are used as benchmarks. We will also discuss a future work on Jacobian-Free-Newton-Krylov DG methods for nonlinear PDEs.

ON OPTIMALLY TOPOLOGICALLY TRANSITIVE ORBITS

Francis C. Motta, Patrick D. Shipman, and Bethany Springer
Colorado State University

We introduce a function $E : \{\gamma_x | x \in M\} \to \mathbb{R} \cup \{\infty\}$ on the orbits of a discrete-time dynamical system, $\Phi(x) : M \to M$ defined on a compact manifold, which we interpret as a measure of an orbit's topological transitivity. Motivated by phyllotactic patterns and problems in mixing, we study the family of dynamical systems $R_\theta : [0, 1) \to [0, 1)$ ($\theta \in (0, 1)$) defined by $R_\theta(x) = (x + \theta) \mod 1$. Utilizing a recursive formula derived from the three-distance theorem, we compute exact values of $E(\{R^t_\theta(x)|t \in \mathbb{N}\}, x \in [0, 1))$, for various choices of θ. We then study this measure of topological transitivity on transitive orbits of Bernoulli shift maps defined on sequences over finite alphabets.

STABILITY AND PARALLEL PERFORMANCE ANALYSIS OF OBLIQUE QR FACTORIZATION ALGORITHMS

Bradley Lowery
University of Colorado, Denver

We study parallel QR factorization algorithms, where the columns of the Q factor are orthogonal with respect to an oblique inner product. The oblique inner product is defined by a symmetric positive definite matrix, A. We present new stability analysis for existing algorithms as well as a new, stable, communication avoiding algorithm. We analyze both the loss of orthogonality and representativity and develop test cases to exhibit different behavior on the bounds. The numerical experiments show the bounds tight. We find that loss of orthogonality, even for the most stable algorithms, may at worst be proportional to the condition number of A. However, this depends greatly on the column space of the matrix.

Finally, we consider the parallel performance of the algorithms. The computation of a matrix-vector and matrix-matrix product with A is the overhead when comparing the cost to QR factorizations schemes with Euclidean inner product. To demonstrate the two extremes, we consider the cases where A is dense and when A is sparse, specifically when A is tridiagonal.

USING A ROBIN BOUNDARY CONDITION WITHIN DOMAIN DECOMPOSITION TO SOLVE ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

Bradley McCaskill
University of Wyoming

In this presentation, we develop a non-overlapping iterative domain decomposition technique to solve an elliptic boundary value problem. The interface condition that is used within the iteration is a Robin boundary condition, which would allow solving each sub-domain independently. This way, the iteration is performed only at the interface level. Each local problem is solved using some type of finite volume procedure. Several numerical experiments are discussed to illustrate the important features of the procedure.

AFTERNOON SESSION I

Modeling Contest, Problem A

COOKING THE ULTIMATE BROWNIE: INTRODUCTION TO THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS

Nathan Pelc
Collaborators: Jack Lepird and Austin Howard
United States Air Force Academy

Nobody likes burnt brownies, yet common rectangular brownie pans create burnt corner brownies and undercooked center brownies. Conversely, circular pans use oven area inefficiently, and still cook the outer edges quicker. In the recent COMAP MCM competition, one of the problems required modeling the outer temperature distribution of various basic brownie pans, and optimizing a pan in order to obtain uniform temperatures and maximum amounts of brownie. We, however, extended this problem to three dimensions. Therefore, our research focused on three issues: first, to accurately model the three-dimensional temperature distributions of various arbitrarily shaped brownie pans; second, to create a brownie pan that minimizes temperature differences throughout the brownie; and finally, to maximize the number of such brownie pans that can fit into an oven. To solve the problem, we used a finite element analysis (FEA) and forward difference approximation in order to numerically solve the 3D heat equation for arbitrarily shaped pans. We also created an optimization of that model to minimize the temperature differences throughout the brownie by adding and removing brownie to locations that were too hot or too cold, respectively. Finally, we also used a system of homogeneous ordinary differential equations (ODEs) to simulate the heat flow within an oven and optimize for the maximum amount of edible brownie cooked per batch.

Modeling Contest, Problem A

THE ULTIMATE BROWNIE: A GUIDE TO MAKE THE ULTIMATE BROWNIE PAN

Danny Dauwe, Taylor Klotz, and Jessica Gronski
University of Colorado, Colorado Springs

We will discuss this year's MCM (mathematics contest in modeling) problem A: *The Ultimate Brownie Pan.* The goal of the contest was to design a brownie pan shape that is not only optimized to provide uniform heating throughout the pan contents, but also maximizes the number of pans that can fit inside an oven at one time. Our first, simple model, explores the optimal packing of a few two-dimensional brownie pan shapes inside of an oven, and then using the heat equation we determine each pan shape's efficiency at distributing heat throughout the brownie in the pan. We will justify our choice of pan shape by describing how our pan shape satisfies both optimal heating and optimal packing. We will also discuss a weight problem we created that combines both the packing problem and the heat distribution problem described above that should explain how one might attain a better pan shape. Furthermore we will also discuss some drawbacks of our models, as well as proposing some suggestions for potentially better models that we did not have time to consider due to the restrictions of the contest. Questions, further suggestions, and discussion is encouraged after we present our results and ideas.

STORING CYCLES IN NEURAL NETWORKS WITH DELAYED COUPLING

Chuan Zhang, Gerhard Dangelmayr, and Iuliana Oprea
Colorado State University

Cyclic patterns of neuronal activity are ubiquitous in neural systems of almost all animal species. To elucidate the underlying mechanisms for the storage and retrieval of cyclic patterns in neural networks is fundamentally important for understanding the origin of rhythmic movements. In this presentation, we summarize our investigations in the storage and retrieval of binary cyclic patterns in neural networks using the pseudoinverse learning rule. The presentation is organized into three parts. First, we show that all cyclic patterns satisfying the transition conditions can be successfully stored and retrieved, and the cyclic patterns satisfying the same transition condition can be stored in the same network, and retrieved with appropriately selected initial conditions. Next, we show how the subspace structures of the vector space spanned by the row vectors of the cyclic patterns determine the topology of the networks constructed from these cyclic patterns. Last, we show that transitions from fixed points to attracting limit cycles (cyclic patterns) are multiple saddle-node bifurcations on limit cycles.

THE FLAG OF BEST FIT AS A REPRESENTATIVE FOR A COLLECTION OF LINEAR SUBSPACES

Tim Marrinan
Colorado State University

We propose a flag of best fit, called the flag mean, as a representative for a collection of linear subspaces. The flag mean algorithm determines a nested sequence of vector spaces that best represents the data with respect to a natural, geometrically driven, optimization criterion. A special case of the flag mean can be directly compared to the commonly used Karcher mean for data living on a Grassmannian manifold. When the Grassmann data points are close together, the flag mean and the Karcher mean provide comparable solutions. The flag mean has a closed form solution that resolves the convergence issues of the Karcher mean, and the algorithm can be run with relatively less computation.

These advantages are demonstrated first on synthetic problems designed to illustrate the differences between the means, and then on challenging real-world clustering and exemplar selection problems.

WEIGHTED STOCHASTIC BLOCK MODEL

Christopher Aicher, Abigail Z. Jacobs, and Aaron Clauset
University of Colorado, Boulder

A popular probabilistic model of community structure in complex networks is the stochastic block model, where each vertex belongs to one of several stochastically equivalent 'blocks' and edges exist or do not with probabilities that depend only on the block memberships of the connecting vertices. The classic stochastic block model is restricted to binary (Bernoulli) networks, in which edges are unweighted. In practice, however, most binary networks are produced after applying a threshold to a weighted relationship, which destroys potentially valuable information

We generalize this model to the important case of edge weights drawn from an exponential family distribution. This *weighted stochastic block model* (WSBM) includes as special cases most standard distributional forms, and thus allows us to use weighted relations directly in recovering latent block structure. Handling these general weight distributions presents several technical difficulties for model estimation, which we solve using a Bayesian approach.

Because of the large parameter space, model estimation is a non-trivial problem. We introduce a variational algorithm to efficiently approximate the model's posterior distribution in dense graphs and a scalable belief propagation algorithm for sparse graphs.

In numerical experiments on edge-weighted networks,our weighted stochastic block model substantially outperforms other techniques in correctly recovering latent block structure, including the common approach of first applying a single threshold to all weights and then applying the classic stochastic block model. We also apply the model to a network of human microbiome sample similarities showing potential application.

This model enables the recovery of latent structure in a broader range of network data than was previously possible.

AFTERNOON SESSION II

AUTOREGULATION OF RETINAL BLOOD FLOW IN PATIENTS WITH GLAUCOMA

Victoria Gershuny
University of Colorado, Boulder

Glaucoma is the second largest cause of blindness worldwide and causes optic nerve damage which leads to irreversible vision loss. Elevated intraocular pressure (IOP) is the only treatable aspect of glaucoma even though there is evidence impaired autoregulation might play a significant role in the physiology of the disease. Autoregulation is the maintaining constant blood flow over a wide range of pressures, via different factors including metabolic, myogenic, carbon dioxide concentration, and sheer stress. In order to learn about the effects of each mechanism and especially tissue levels of carbon dioxide, the retinal vascular network is modeled with a series of time-dependent differential equations that are solved at steady state. The results show that inclusion of all of the mechanisms gives an autoregulatory range typically seen in the retina. Increases in IOP decrease the retinal ability to maintain a constant flow rate with decreased pressure. Of all the mechanisms, carbon dioxide concentrations in the tissue and metabolic responses have the most significant effect on the autoregulation.

HIV-1 GAG TRIMERIZATION NEAR CELL MEMBRANE: MATHEMATICAL MODELS AND NUMERICAL SIMULATIONS

Roberto Munoz-Alicea
Colorado State University

HIV is an infectious disease that takes away many peoples lives every year. Gag protein is a major structural component of HIV. The mechanisms of Gag protein trafficking and multimerization in the host cell are still poorly understood. We develop and analyze mathematical models and numerical algorithms for Gag protein dynamics. In particular, we model the trimerization of Gag protein near the cell plasma membrane. Linear stability analysis and numerical simulation results will be discussed.

A LOOK AT THE POTENTIAL FUNCTION FOR MULTI-GRAPHS

Casey Moffatt
University of Colorado, Denver

A sequence of nonnegative integers $\pi = (d_1, d_2, ..., d_n)$ is *graphic* if there is a (simple) graph G with degree sequence π. In this case, G is said to *realize* or be a *realization of* π. Given a graph H, a graphic sequence π is *potentially H-graphic* if there is some realization of π that contains H as a subgraph. In 1991, Erdős, Jacobson and Lehel posed the following, which can be viewed as a degree sequence analogue to the classical Turán problem, "Determine the minimum integer $\sigma(H, n)$ such that every n-term graphic sequence with sum at least $\sigma(H, n)$ is potentially H-graphic." The exact value of $\sigma(H, n)$ has been determined for a number of specific classes of graphs (including cliques, cycles, complete bigraphs and others). In this talk, we will discuss the extension of this *potential function*, $\sigma(H, n)$, where H is a (loopless) multi-graph.

NUMERICAL PRECONDITIONING FOR HOMOTOPY CONTINUATION

Steven Ihde[1]
Collaborators: Dan Bates[1] and Jon Hauenstein[2]
Colorado State University[1]
North Carolina State University[2]

While solving polynomial systems using homotopy continuation, it is often the case where systems have a larger number of potential solutions than actual. Some of these potential solutions may run off to infinity as we track the path to them with homotopy continuation. Our goal is to precondition such systems in order to remove these points at infinity. In this way, we track fewer paths and the paths are better conditioned. An algorithm for preconditioning these systems will be presented.

CLOUD CONVEYOR SYSTEM

Timothy Morris
Collaborators: Shivakumar Sastry, Michael S. Branicky, P.S. Shastry, Michael Ferrara, and Ellen Gethner
University of Colorado, Denver

The cloud conveyor system is a cyber-physical system for the automated transportation of objects through a grid of tracks. Each track has a single cart which traverses the track and returns at a constant speed. When vertically moving carts and horizontally moving carts are at the same grid location they may transfer their contents. We analyze conditions to guarantee that an entity may be conveyed through the system as well as a technique to find the shortest route for such an entity to take through the system.

AFTERNOON SESSION III

VALIDATING FALCON TELESCOPE NETWORK SIMULATIONS WITH PRELIMINARY DATA

Brandon A. Mueller
United States Air Force Academy

With more nations building and launching satellites, space has become more congested, contested, and competitive. Space situational awareness is a vital component of U.S. national security due to the role space and cyberspace play in our military operations. This requires a migration from catalog orbital maintenance to full characterization and assessment of space objects. The U.S. Air Force Academy Falcon Telescope Network(FTN), a global network of small aperture telescopes, allows the development of new techniques and algorithms for satellite characterization and data fusion in assessing potential threats in the space domain. This talk presents FTN modeling in conjunction with preliminary data to demonstrate the network's research capability in non-resolvable space object identification. Specifically, simulation data from a photometric modeling tool will be compared to real-world observations for the NASA satellite Fastsat.

ASSIMILATION OF FIRE PERIMETER DATA INTO THE FIRE SPREAD MODEL SFIRE COUPLED WITH THE WRF MODEL

Volodymyr Y. Kondratenko, Jonathan D. Beezley, Jan Mandel, and Adam K. Kochanski
University of Colorado, Denver

Fire simulation starts from an ignition point, and when fire perimeter data are assimilated, the state of the fire spread model is changed to become closer to the data. This, however, presents a difficulty in coupled fire-atmosphere models. The fire has a very strong effect on the atmosphere and changes in atmospheric state due to the fire take time to develop, so the existing atmospheric circulation is no longer compatible with the modified fire. Moreover, linearized changes to the atmospheric state have no hope of establishing the properly changed circulation in a physical balance. We have recently developed a technique for fire ignition from perimeter data, which goes back in time and replays an approximate fire history to allow the proper atmospheric circulation patterns to develop. Here, we extend this technique to the assimilation of a fire perimeter into a developed fire state with an established atmospheric state. The SFIRE model uses the level set method to simulate the fire spread. Our data assimilation approach takes advantage of the manipulation of the fire state through level set functions, which is much easier than manipulating the fire areas directly.

APPLICATION OF NUMERICAL ALGEBRAIC GEOMETRY TO GEOMETRIC DATA ANALYSIS

Daniel Bates[1], Brent Davis[1], Michael Kirby[1], Justin Marks[2], and Chris Peterson[1]
Colorado State University[1]
Air Force Institute of Technology, Wright Patterson Air Force Base[2]

For two fixed vector spaces V and W of a finite-dimensional vector space over $\mathbb{R}$ define

$$\cos(\theta(V,W)) = \max_{v\in V, w\in W}\{v\cdot w | \|v\| = \|w\| = 1\}.$$

Note that

$$\theta(\mathcal{V},\mathcal{W}) = \min_{v\in \mathcal{V}, w\in \mathcal{W}}\{\cos^{-1}(v\cdot w) | \|v\| = \|w\| = 1\}$$

is called the first principal angle between subspaces $\mathcal{V}$ and $\mathcal{W}$.

Let $\{\mathcal{V}_1,\ldots,\mathcal{V}_k\}$ be a set of subspaces of a finite-dimensional vector space $\mathcal{V}$ over $\mathbb{R}$. Consider the problem

$$\max_{l\in\mathcal{V}} \sum_{i=1}^{k} \cos(\theta(\mathcal{V}_i, l))$$

where l is a 1-dimensional subspace of $\mathcal{V}$.

The problem originated in the setting of geometric data analysis. We reformulate the problem and solve it by using tools from numerical algebraic geometry and homotopy continuation.

CONSERVATIVE FLUX FROM THE CONTINUOUS GALERKIN FEM

Lawrence Bush
University of Wyoming

Many applications in subsurface flow require flux conservation. It is well known that the simple continuous Galerkin Finite Element Method will not produce a locally conservative flux because the velocity is not treated as a primary variable. Naive calculation of the velocity from the gradient of the pressure solution will most likely result in a highly discontinuous velocity field. This direct calculation is one way of post-processing a Galerkin FEM solution to derive a velocity. A simple post-processing that produces a conservative flux is desired.
Our present work has developed a simple post-processing for continuous Galerkin FEM solutions resulting in a conservative flux. The post-processing requires solving a small system on each element. The technique will be presented along with applications to two-phase flow simulations that may include multi-scale behavior.

TESTING THE EFFECTS OF ALTITUDE ON ROCKET MOTOR PERFORMANCE

Ryan Hartz
United States Air Force Academy

This presentation will detail the progress and current results of research centered on measuring the effects of high altitude on rocket motor performance. Rocket motors produce greater thrust at higher altitudes versus lower altitudes due to the decrease in ambient pressure with increasing altitude. A formula which predicts the thrust increase is equal to the decrease in ambient pressure times the cross sectional area of the throat of the nozzle. The research aims to measure the increase in the rocket motor impulse and comparing it to the increase predicted by this formula.

This is accomplished using a high altitude testing site located in Colorado, and a low altitude testing site located in Louisiana and employing hobbyist rocket motors. This research is ongoing, but promising results have been made and discrepancies between the manufacture's rocket motor specifications and measured results have been found.